CREEP ANALYSIS OF ORTHOTROPIC SHELLS

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CERTIFICATE

This is to certify that the thesis entitled "CREEP ANALYSIS OF ORTHOTROPIC SHELLS" is a record of work carried out under my supervision and that it has not been submitted elsewhere for a degree.

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SYNOPSIS
of the
Dissertation on
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A method of creep analysis of orthotropic circular cylindrical and spherical shells subjected to axisymmetric loads has been developed in the present work. A general study of creep behaviour of cylindrical and spherical shells subjected to uniform internal pressure has been conducted for a wide range of the values of anisotropy coefficients and creep law exponent. Importance of thermal stresses has been illustrated by a particular example of cylindrical shell. Analysis includes determination of stress redistribution, strain rates, stationary state stresses.

Application of reference stress technique has been extended to the analysis of shells.

Details of finite difference technique as applied to the present problem are discussed and suggestions for efficient computer use are included.

LIST OF SYMBOLS

a = radius of shell

$A_{xx}, A_{\theta\theta}, A_{x\theta}$ = anisotropy coefficients of creep

E_1, E_2, E_ν = elastic constants of material

E^* = $E_2 - E^{2}/E_1$

h = thickness of shell

k = characteristic parameter of shell

$$= \left(3 \frac{E^*}{E_1} \frac{a^2}{h^2} \right)^{1/4}$$

K_x = change of curvature $= - \dfrac{d^2 w}{dx^2}$

L^2 = operator : $\dfrac{d^2}{d\phi^2} + \cot \phi \dfrac{d}{d\phi} - \dfrac{E_2}{E_1} \cot^2 \phi$

m = creep law exponent

M = bending moment per unit length

m_x, m_θ, m_ϕ = non-dimensional bending moment

$$= \frac{M_x}{\sigma_o h^2}, \frac{M_\theta}{\sigma_o h^2}, \frac{M_\phi}{\sigma_o h^2}$$

N = stress resultant per unit length

n_x, n_θ, n_ϕ = non-dimensional stress resultant

$$= \frac{N_x}{\sigma_o h}, \frac{N_\theta}{\sigma_\theta h}, \frac{n_\phi}{\sigma_o h}$$

p = internal pressure

Q = lateral shear force per unit length

q	=	non dimensional shear force per unit length of cylindrical shell
	=	$\frac{2\ k\ Q}{\sigma_o\ h}$
S	=	non dimensional shear force per unit length for spherical shell
	=	$\frac{2Q\ k^2}{\sigma_o\ h}$
t	=	time
T	=	Temperature
T_o	=	unit temperature
T'	=	non dimensional temperature
	=	$\frac{T}{T_o}$
u	=	displacement of an element of cylindrical shell parallel to its axis
U_1	=	$\sum_e^{m-1} \left(\Sigma_x + \frac{E_1 A_{x\theta} + E_\nu A_{\theta\theta}}{E_1 A_{xx} + E_\nu A_{x\theta}} \Sigma_\theta \right)$ for cylindrical shell
	=	$\sum_e^{m-1} \left(\Sigma_\phi + \frac{E_1 A_{x\theta} + E_\nu A_{\theta\theta}}{E_1 A_{xx} + E_\nu A_{x\theta}} \Sigma_\theta \right)$ for spherical shell
U_2	=	$\sum_e^{m-1} \left(\Sigma_\theta + \frac{E_2 A_{x\theta} + E_\nu A_{xx}}{E_2 A_{\theta\theta} + E_\nu A_{x\theta}} \Sigma_x \right)$ for cylindrical shell

$U_2 = \sum_{e}^{m-1} \left(\Sigma_\theta + \frac{E_2 A_{x\theta} + E_\nu A_{xx}}{E_2 A_{\theta\theta} + E_\nu A_{x\theta}} \Sigma_\phi \right)$

for spherical shell

$U_1' = U_1 \, P(T)$

$U_2' = U_2 \, P(T)$

v = displacement along tangent to spherical shell meridian

V = Rotation of tangent to shell meridian (for spherical shell)

w = radial displacement of shell

z = distance across shell wall

α = a constant

α_1, α_2 = coefficients of thermal expansion for the axial and circumferential directions of cylindrical shell

$\gamma = -\frac{d\lambda_\theta}{d\eta}$ (for cylindrical shell)

Δ = distance between two adjacent nodes

η = non dimensional length parallel to axis of cylindrical shell

$= \sqrt{2}\, k \frac{x}{a}$

ξ = non dimensional distance across wall thickness

$= 2z/h$

ϵ = strain

ϵ_o = unit strain

$= \frac{\sigma_o}{E^*}$

ϵ_e = effective strain

λ = reduced strain

= $\dfrac{\epsilon}{\epsilon_o}$

σ = principal stress

σ_o = unit stress

= $\alpha \dfrac{pa}{2h}$

σ_e = effective stress

Σ = non-dimensional stress measure

= $\dfrac{\sigma}{\sigma_o}$

τ = non-dimensional time measure

ϕ' = fast neutron flux normalised with respect to 10^{13} n/cm^2-sec

Θ = reduced rotation of tangent to shell meridian (for spherical shell)

= $\dfrac{V}{\epsilon_o}$

SUBSCRIPTS

c denotes creep component

el denotes elastic component

x, θ, z refer to axial, circumferential and radial components (for cylindrical shell)

ϕ, θ, z	refer to meridional, circumferential and radial components (for spherical shell)
ss	denotes steady state

SUPERSCRIPTS

$(\dot{\ })$	denotes differentiation with respect to non-dimensional time
$(\bar{\ })$	denotes value associated with reference stress

CONTENTS

CHAPTER I

INTRODUCTION

A large number of modern industries involve systems having cylindrical and spherical shells subjected to high temperature and pressure. Nuclear, steam and gas turbine power plants, chemical industries, supersonic transport are a few of such cases where the design of cylindrical and spherical shells is of extreme importance. It has been found that creep phenomenon plays a dominating role in the process of deformation of such shells. One of the primary aims of a designer of equipment subjected to creep, is to offset the possibilities of failure by excessive deformation. Dimensional changes with time may lead to serious problems and ultimately to failure of the system.

Creep deformation of shells, made of isotropic materials, has been investigated by a large number of researchers. The assumption that a material starts off, and remains, in an isotropic state cannot be always justified. Materials become anisotropic during working and heat treatment processes and continue to do so under load. Creep analysis of structural members made of orthotropic materials has been limited to thin and thick cylinders. In the present work creep behaviour of shells, made of orthotropic materials, has been investigated.

Reference stress method for estimation of creep deformation has been applied to simple structures like beams and cylinders by many investigators. In the present work it has been attempt-

to investigate the applicability of reference stress techniques for creep analysis of shells.

1.1 REVIEW OF PAST WORK

1.1.1 Creep Under Constant Uniaxial Stress :

A tensile specimen under constant stress will deform with time. This deformation depends on three main parameters, stress, time and temperature. The most general creep equation is therefore

$$\epsilon_c = f\ (\sigma,\ t,\ T) \tag{1.1}$$

A useful first approximation is to limit this general function to a law of the form

$$\epsilon_c = f_1\ (\sigma)\ \ f_2(t)\ \ P(T) \tag{1.2}$$

The separation of functions $f_1\ (\sigma)$ and $f_2\ (t)$ is common in most of work on creep and appears to be generally acceptable for the purpose of calculations on components.

The function $f_1\ (\sigma)$ has been chosen in many different ways. Kennedy gives a full summary(1)*. The most common forms are given below.

$$\begin{array}{lll} \text{Norton} & f_1(\sigma) = & L\ \sigma^m \\ \text{Mcvetty} & f_1(\sigma) = & A\ \text{Sinh}\ (\sigma / \sigma_o) \\ \text{Garofalo} & f_1(\sigma) = & A\ \ \text{Sinh}\ (\sigma/\sigma_o)^m \\ \text{Johnson} & f_1(\sigma) = & D_1\ \sigma^{m_1} + D_2\ \sigma^{m_2} \end{array} \tag{1.3}$$

where L, A, D_1, D_2, m, m_1, m_2 are material constants.

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* Number within brackets refers to reference number.

The most commonly used function is the power law attributed to Norton. The reason for its popularity is its simplicity in stress analysis.

A large number of alternative expressions are suggested for description of function f $(t)^{(1)}$. In practice, the time dependence of creep in a complex alloy can be established only by curve fitting of experimental data. In such an emperical approach complex actions associated with shape of $f_2(t)$ are lumped together into one simple explicit time function. The following time functions are of practical use.

$$\left.\begin{aligned} &\text{Andrade} \quad f_2(t) = (1+ bt^{1/3}) \exp (k_1 t) - 1 \\ &\qquad\qquad\qquad \simeq bt^{1/3} \\ &\text{Bailey} \quad f_2(t) = F\, t^n \\ &\text{Graham and Walles } f_2(t) = \sum_i a_i\, t_i^{n_i} \end{aligned}\right\} \quad (1.4)$$

b, k_1, F, n, a_i, n_i are constants.

Temperature has two effects on creep strain rate. Firstly, a change in temperature has immediate effect on material constants. Secondly, temperature encourages changes in material structure. Some of important relations are reproduced here from reference 1.

$$\left.\begin{aligned} \epsilon_c &= A \exp (- Q/RT) \\ \epsilon_c &= A \left[t \exp (-Q/RT)\right]^n \\ \epsilon_c &= A \quad T \exp (-Q/RT) \end{aligned}\right\} \quad (1.5)$$

where Q is the activation energy, R is the Boltzman's constant and A is a constant.

1.1.2 Creep Under Multiaxial Stress :

The first step in creep analysis under multiaxial stress system is to determine the combined effect of a multiaxial stress system to produce a certain magnitude of strain. This has led to acceptance of twin quantities, equivalent stress and equivalent strain increment, defined, in general as

$$\begin{aligned} \sigma_e &= \sigma_e\,(\sigma_x, \sigma_y, \sigma_z) \\ \Delta\epsilon_e &= \Delta\epsilon_e\,(\Delta\epsilon_x, \Delta\epsilon_y, \Delta\epsilon_z) \end{aligned} \tag{1.6}$$

These quantities are usually chosen to reduce to correct values of stress and strain increment in the uniaxial case. For example, an m-power relationship is $\Delta\epsilon_e = L\,\sigma_e^m\,\Delta t$. By analogy with plasticity laws effective stress σ_e is commonly assumed to take the same form as von-Mises yield function. For isotropic materials this is given by

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2\right]^{1/2} \tag{1.7}$$

The next step is to determine principal strain increments. Extending the analogy with plasticity, equations similar to Prandtl-Reuss flow rule can be written as given below[2].

$$\frac{d\epsilon_{xc}}{dt} = f_1\,(\sigma_e)\left(\sigma_x - \frac{\sigma_y + \sigma_z}{2}\right)\frac{1}{\sigma_e}\,P\,(T) \text{ etc.} \tag{1.8}$$

Bhatnagar and Gupta[3] derived constitutive equations for creep behaviour of orthotropic material. The form postulated is similar to the one for isotropic material. Effective stress and relation ~~behaviour~~ between strain rate and stress are

$$\sigma_e = \frac{1}{\sqrt{2}}\left[F\ (\sigma_x - \sigma_y)^2 + G\ (\sigma_y - \sigma_3)^2 + H\ (\sigma_x - \sigma_z)^2\right]^{\frac{1}{2}} \qquad (1.9)$$

$$\frac{d\epsilon_{xc}}{dt} = \frac{f_1(\sigma_e)}{2\,\sigma_e}\left[(F+H)\,\sigma_x - F\,\sigma_y - H\,\sigma_z\right] \text{ etc.} \qquad (1.10)$$

where F, G, H depend on material anisotropy.

The equation (1.9) is based on assumption that creep behaviour is same in tension and compression.

Bhatnagar and Gupta[4] used these relations for creep analysis of thick walled cylinders.

1.1.3 Creep Analysis of Structures :

A method for the determination of stationary creep of pressure vessel closures with constant stress rates using membrane theory of shells and power creep law was developed by Gemma, Rowe and Spahl[5].

The state of deformation and stress in a circular cylindrical shell subjected to uniform lateral pressure was analysed by Bienick and Freudenthal[6]. A velocity field $\dot{\omega}$ of deformed shell was assumed with some parameters which were determined with the aid of condition that power

$$\int_0^{l}\left[\frac{n}{n+1}\left(M_x\frac{dk_x}{dt} + \frac{N_\theta\, d\epsilon_\theta}{dt}\right) - p\,\dot{\omega}\right]dx \quad \text{becomes minimum.}$$

Some approximations of the type

$$\frac{d\epsilon_\theta}{dt} = f\ (t, \tau)\ \sigma_\theta^{\,m} \quad \text{were used.}$$

Sankarnarayanan[7] presented a method for stationary state creep analysis of circular cylindrical shell under axially

symmetric load. The analysis was based on "square" interaction criterion between generalized stresses, namely, circumferential force and axial bending moment. It was assumed that creep strain rate is a product of power function of a generalized stress and a function of time.

A simplified theory of stationary state creep of shells has been described by Rabotnov[8]. The real shell was replaced by a two layer model. A constant stress was assumed in each of these layers. Thickness of layers and distance between them was so selected that behaviours of the real shell and its two layered model were similar in a moment-free state of stress and also in case of cylindrical bending.

The methods described above may be used for stationary state analysis only and hence they do not give an estimate of redistribution of stresses.

Penny[9,10] developed a method for calculating creep deformation in spherical shell (with axisymmetric openings) and clamped circular cylindrical shell. In this method, which determines the redistribution of stresses also, starting point is the elastic solution. Stress rates and strain rates which depend on instantaneous value of stress can be calculated. Forward integration in time gives stresses after a small interval. Such a process may be repeated till stress rates throughout the structure become low.

A method of analysis of creep bending in circular plates with temperature gradients was presented by Cozzarelli[11]. The analysis was based on a power creep law in which the creep

parameter was expressed as a function of temperature. Results were obtained for plates subjected to radially symmetric loads and temperatures.

1.1.4 Reference Stress :

Deformation rates in some structures which obey power law can be estimated from the results of a single test conducted at a particular value of stress called reference stress.

Marriott and Leckie studied stress redistribution which occurs during creep of a beam in pure bending and a pressurized thick walled cylinder, when elastic and primary creep effects are included. They found that magnitude of effective stress during progressive stages of creep remained constant at some point which was named skeletal point. Both the components studied belong to a special class for which strain distribution is independent of the type of stress-strain relation. In such structures reference stress coincides with stress at skeletal point. The analysis has been described by Marriott in his review paper[12].

Mackenzie[13] showed that reference stress can be established by comparing the m-power solution with that for m = 1. Results were obtained for beams in bending, pressurised thin walled cylinder and sphere, thick walled cylinder and axisymmetrically loaded circular plates. It may be remarked that application of this method is limited to structures for which analytical expressions for stationary state deformation are available.

Sim[14] extended Mackenzie's approach to include the results of numerical solutions. Problem of beam in pure bending and of spinning disc mounted on rigid boss were solved.

Sim[15] described a general method of obtaining reference stress from the dimensionless solutions of creep behaviour of a structure. The method requires stationary state solutions for two different values of creep law exponent. Reference stresses for a spinning disc mounted on a rigid boss and pressurised thick walled cylinder were obtained.

1.2 OBJECTIVES AND SCOPE OF THE WORK

All the earlier investigations into creep of shells assume that material is isotropic. This assumption is not a realistic one. Materials become anisotropic during working and heat treatment. Also materials may become anisotropic when subjected to loads.

In the present work a method of analysis has been developed to find creep deformation of shells made of orthotropic material. Uniaxial creep law of type

$$\frac{d\epsilon_c}{dt} = B\ (t)\ \ P(T)\ \ \sigma^m$$

has been used throughout analysis. The main reason for choosing a power type law is that it has provided fairly accurate results in earlier investigations. At the same time analysis is made much simpler. Moreover, many materials like carbon steel, stainless steel, aluminium alloys, nickel alloys and zirconium alloys have been found to obey such a type of relation[16,17].

A constitutive law of the type given by equation (1.10) has been used to describe stress-strain behaviour in a multiaxial stress field.

Cold working does give rise to difference in creep rates in tension and compression just as it produces preferred orientation. The former can be removed by mild annealing. Hence it is perfectly possible, both in principle and practice, to have a material with identical creep characteristics in tension and compression and thereby satisfy assumptions inherent in equation (1.10). Zirconium alloys, for example, have been reported to follow a creep law of this type[17].

Due to lack of data, change of anisotropic behaviour during creep has been neglected in the analysis.

It has been shown that reference stress technique, which greatly reduces the number of creep tests required to establish creep behaviour of a structure, can be applied to shells also.

For the class of materials described earlier the following problems have been solved.

(i) Determination of stress redistribution, strain rates, steady state stresses and reference stress for (a) circular cylindrical shell with either fixed or simply supported edges (b) clamped spherical shell subjected to uniform internal pressure.

This problem has been solved for a wide range of the values of stress exponent and anisotropy coefficients.

(ii) Determination of stress redistribution, strain rates and steady state stresses in a clamped cylindrical shell subjected to a uniform pressure and axisymmetric linearly varying temperature field.

This problem has been solved for a shell operating in fast neutron flux and made of zircaloy-2.

All the results are presented in form of graphs. All the parameters associated with analysis are non-dimensional. However, the results presented are confined to a particular value of dimensionless length for cylindrical shells and of subtended meridian angle and radius to thickness ratio in case of spherical shells.

Computer programs have been written for numerical evaluation of stresses and strains at different times during creep.

CHAPTER II

MATHEMATICAL FORMULATION AND METHOD OF SOLUTION

In this Chapter equations for elastic and creep analysis of shell problems stated in Chapter I have been derived and methods of solution are developed.

2.1 ASSUMPTIONS

(i) The material is homogeneous and orthotropic. The axes of orthotropy coincide with the direction of principal stresses.

(ii) Power creep law stated below is applicable to creep deformation in uniaxial stress field.

$$\frac{d\epsilon_c}{dt} = B(t) \quad P(T) \, \sigma^m \tag{2.1}$$

(iii) Von-Mises yield criterion and Prandtl-Reuss flow rate for orthotropic material[3] are applicable. Stress-strain rate relations of type given by equation (1.10) are used. They are based on assumptions that creep behaviour in tension is similar to that in compression and creep strains do not cause any change in volume. Also creep is assumed to occur in any non-zero stress field.

(iv) Displacements are small.

(v) Normals to the middle surface are conserved as such during deformation.

(vi) Shell is thin so that z/a can be neglected in comparison with unity. This, in conjunction with (v), forms Kirchoff-Love assumptions.

(vii) Structure is in elastic state at time equal to zero.

This assumption is valid for many practical cases.

(viii) Load is constant.

2.2 ELASTICITY EQUATIONS FOR CIRCULAR CYLINDRICAL SHELL SUBJECTED TO UNIFORM INTERNAL PRESSURE

Sign convention is shown in Figure.2.1.

2.2.1 Stress-strain Relations :

For an orthotropic material stress-strain relations are[21]

$$\begin{aligned} \sigma_x &= E_1 \epsilon_{xel} + E_\nu \epsilon_{\theta el} \\ \sigma_\theta &= E_\nu \epsilon_{xel} + E_2 \epsilon_{\theta el} \end{aligned} \qquad (2.2)$$

The above equation may be written in non-dimensional form as

$$\begin{aligned} \Sigma_x &= \frac{E_1 \lambda_{xel} + E_2 \lambda_{\theta el}}{E^*} \\ \Sigma_\theta &= \frac{E_2 \lambda_{\theta el} + E_\nu \lambda_{xel}}{E^*} \end{aligned} \qquad (2.2a)$$

2.2.2 Strain-displacement Relations :

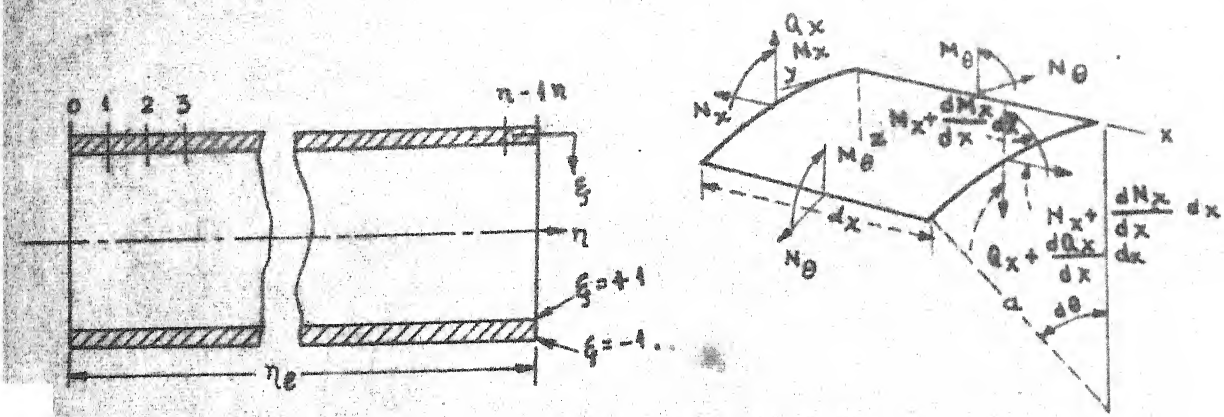
These relations are based on Kirchoff's assumptions[18]

$$\begin{aligned} \epsilon_x &= \frac{du}{dx} - z\,\frac{d^2 w}{dx^2} \\ \epsilon_\theta &= -\frac{w}{a} \end{aligned} \qquad (2.3)$$

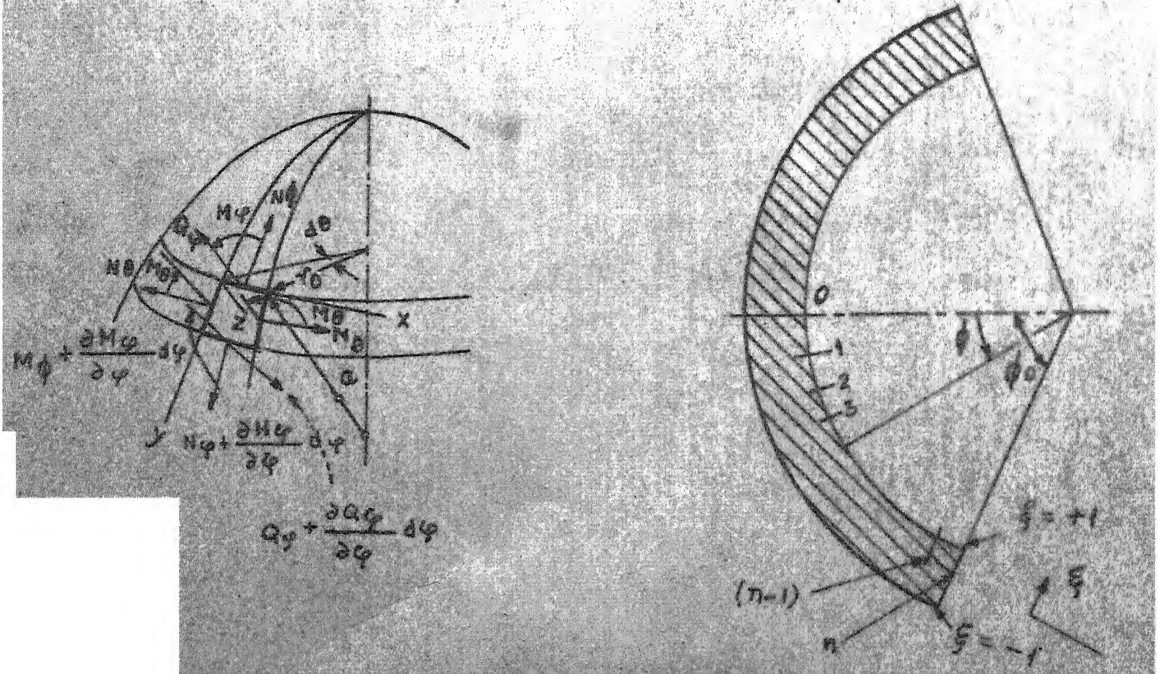
2.2.3 Stress-resultants and Stress Couples :

Assuming Love's first hypothesis

$$N_x = \int_{-h/2}^{h/2} \sigma_x \, dz$$



(a) CYLINDRICAL SHELL



(b) SPHERICAL SHELL

FIG. 2.1 COORDINATE SYSTEM

(vii) Structure is in elastic state at time equal to zero.

This assumption is valid for many practical cases.

(viii) Load is constant.

2.2 ELASTICITY EQUATIONS FOR CIRCULAR CYLINDRICAL SHELL SUBJECTED TO UNIFORM INTERNAL PRESSURE

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For an orthotropic material stress-strain relations are [21]

$$\sigma_x = E_1 \epsilon_{xel} + E_\nu \epsilon_{\theta el}$$
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The above equation may be written in non-dimensional form as

$$\Sigma_x = \frac{E_1 \lambda_{xel} + E_2 \lambda_{\theta el}}{E^*}$$
$$\Sigma_\theta = \frac{E_2 \lambda_{\theta el} + E_\nu \lambda_{xel}}{E^*} \qquad (2.2a)$$

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$$\epsilon_x = \frac{du}{dx} - z \frac{d^2 w}{dx^2}$$
$$\epsilon_\theta = -\frac{w}{a} \qquad (2.3)$$

2.2.3 Stress-resultants and Stress Couples :

Assuming Love's first hypothesis

$$N_x = \int_{-h/2}^{h/2} \sigma_x \, dz$$

If the ends of shell are closed,

$$N_x = \frac{pa}{2}$$

Substituting equations (2.2) and (2.3) in the above equation,

$$\frac{du}{dx} = \frac{\sigma_o}{E_1} \frac{1}{\alpha} + \frac{E_\nu}{E_1} \frac{w}{a} \tag{2.4}$$

Based on Love's hypothesis,

$$N_\theta = \int_{-h/2}^{h/2} \sigma_\theta \, dz \; , \; M_x = \int_{-h/2}^{h/2} \sigma_x \, z \, dz \; , \; M_\theta = \int_{-h/2}^{h/2} \sigma_\theta \, zdz$$

Substituting equations (2.2) through (2.4), the above equations are written in the following non-dimensional form.

$$n_\theta = \frac{E_\nu}{E_1} \frac{1}{\alpha} - \frac{w}{a} \frac{1}{\epsilon_0} \tag{2.5}$$

$$m_x = -\frac{1}{2} \frac{d\gamma}{d\eta} \left(\frac{E_1}{3E^*}\right)^{1/2} \tag{2.6}$$

$$m_\theta = \frac{E_\nu}{E_1} m_x \tag{2.7}$$

Substituting equation (2.5) in equation (2.3), reduced strains are written as

$$\lambda_x = \frac{E_2}{E_1} \frac{1}{\alpha} - \frac{E_\nu}{E_1} n_\theta - \frac{d\gamma}{d\eta} \left(\frac{3E^*}{E_1}\right)^{1/2} \tag{2.8}$$

$$\lambda_\theta = n_\theta - \frac{E_\nu}{E_1} \frac{1}{\alpha} \tag{2.9}$$

2.2.4 Equilibrium Equations :

Equilibrium equations as given in reference 18 are

$$
\left.
\begin{aligned}
\frac{dN_x}{dx} &= 0 \\
\frac{dQ_x}{dx} + \frac{N_\theta}{a} &= p \\
\frac{dM_x}{dx} - Q_x &= 0
\end{aligned}
\right\} \qquad (2.10)
$$

Second equilibrium equation may be written as

$$
n_\theta = \frac{2}{\alpha} - \frac{dq}{d\eta} \qquad (2.11)
$$

First equilibrium equation and equation (2.4) are similar to each other.

Combining equations (2.5) and (2.6) with the last two equilibrium equations,

$$
\left.
\begin{aligned}
\frac{d^2 q}{d\eta^2} - \gamma &= 0 \\
\frac{d^2 \gamma}{d\eta^2} + q &= 0
\end{aligned}
\right\} \qquad (2.12)
$$

2.3 CREEP EQUATIONS FOR CIRCULAR CYLINDRICAL SHELL SUBJECTED TO UNIFORM INTERNAL PRESSURE

2.3.1 Stress-creep Strain Rate Relation :

Assuming that strain rates depend on state of stress in the structure,

$$\left.\begin{aligned}
\sigma_e &= (A_{xx}\,\sigma_x^{\,2} + A_{\theta\theta}\,\sigma_\theta^{\,2} + 2A_{x\theta}\,\sigma_x\,\sigma_\theta)^{1/2} \\
\frac{d\,\epsilon_{xc}}{dt} &= \sigma_e^{\,m-1}\,(A_{xx}\,\sigma_x + A_{x\theta}\,\sigma_\theta)\,P(T)\,B(t) \\
\frac{d\,\epsilon_{\theta c}}{dt} &= \sigma_e^{\,m-1}\,(A_{\theta\theta}\,\sigma_\theta + A_{x\theta}\,\sigma_x)\,P(T)\,B(t)
\end{aligned}\right\} \qquad (2.13)$$

A_{xx}, $A_{\theta\theta}$, $A_{x\theta}$, which are constants of the material, are called coefficients of anisotropy. A_{xx} is taken equal to unit for the sake of simplicity. $A_{\theta\theta}$ represents the ratio of creep rate of uniaxial specimen with axis parallel to circumferential direction of the shell to the creep rate of uniaxial specimen with axis parallel to longitudinal axis of the shell. $A_{x\theta}$ measures the poisson effect due to creep in longitudinal and circumferential directions.

Dimensionless time measure τ is defined as

$$\left.\begin{aligned}
\tau &= E^{*}\,\sigma_0^{\,m-1}\,P(T)\int_0^t B(P)\,dp \\
&= \frac{E^{*}\,P(T)}{\sigma_0}\int_0^t \sigma_0^{\,m}\,B(P)\,dp
\end{aligned}\right\} \qquad (2.14)$$

The above expression is seen to be equivalent to the ratio of creep strain for uniaxial test at stress σ_0 to elastic strain due to stress σ_0.

Real time values can be calculated from τ by relation

$$t = \left[\frac{(n+1)\;\tau}{KE^{*}\,\sigma_0^{\,m-1}}\right]^{\frac{1}{n+1}} \qquad (2.15)$$

Non-dimensional form of equation (2.13) is

$$\Sigma_e = (A_{xx}\,\Sigma_x^{\,2} + A_{\theta\theta}\,\Sigma_\theta^{\,2} + 2A_{x\theta}\,\Sigma_x\,\Sigma_\theta)^{1/2} \tag{2.16}$$

$$V_1 = \dot{\lambda}_{xc} = \Sigma_e^{\,m-1}\,(A_{xx}\,\Sigma_x + A_{x\theta}\,\Sigma_\theta) \tag{2.17}$$

$$V_2 = \dot{\lambda}_{\theta c} = \Sigma_e^{\,m-1}\,(A_{\theta\theta}\,\Sigma_\theta + A_{x\theta}\,\Sigma_x) \tag{2.18}$$

2.3.2 Stress-resultant Rate and Stress Couple Rate :

$$\left.\begin{aligned} \dot{\epsilon}_x &= \dot{\epsilon}_{xel} + \dot{\epsilon}_{xc} \\ \dot{\epsilon}_\theta &= \dot{\epsilon}_{\theta c} + \dot{\epsilon}_{\theta el} \end{aligned}\right\} \qquad (2.\ldots$$

Equations relating rates of change of stress, strain and displacement are similar to equations (2.2) and (2.3). Combining such relations with equation (2.19),

$$\left.\begin{aligned} \dot{\sigma}_x &= E_1\left(\frac{d\dot{u}}{dx} - z\,\frac{d^2\dot{w}}{dx^2}\right) - E_\nu\,\frac{\dot{w}}{a} - (E_1\,\dot{\epsilon}_{xc} + E_\nu\,\dot{\epsilon}_{\theta c}) \\ \dot{\sigma}_\theta &= -E_2\,\frac{\dot{w}}{a} + E_\nu\left(\frac{d\dot{u}}{dx} - z\,\frac{d^2\dot{w}}{dx^2}\right) - (E_2\,\dot{\epsilon}_{\theta c} + E_\nu\,\dot{\epsilon}_{xc}) \end{aligned}\right\} \tag{2.20}$$

Since structure is subjected to constant load

$$\dot{N}_x = \int_{-h/2}^{h/2} \dot{\sigma}_x\,dz = 0$$

Substitute equation (2.20) in the above relation to get

$$\frac{d\dot{u}}{dx} = \frac{1}{h}\int_{-h/2}^{h/2} \left(\dot{\epsilon}_{xc} + \frac{E_\nu}{E_1}\,\dot{\epsilon}_{\theta c}\right) dz + \frac{E_\nu}{E_1}\,\frac{\dot{w}}{a} \tag{2.21}$$

Love's hypothesis gives

$$\dot{N}_\theta = \int_{-h/2}^{h/2} \dot{\sigma}_\theta \, dz \; ; \quad \dot{M}_x = \int_{-h/2}^{h/2} \dot{\sigma}_x \, z \, dz \; ; \quad \dot{M}_\theta = \int_{-h/2}^{h/2} \dot{\sigma}_\theta \, z \, dz$$

The above relations are non-dimensionalized to the following form by using equations (2.20) and (2.21).

$$\dot{n}_\theta = - E^* \frac{\dot{w}}{a} \frac{1}{\sigma_o} - \frac{1}{2} \int_{-1}^{1} V_2 \, d\xi \tag{2.22}$$

$$\dot{m}_x = - \frac{1}{2} \frac{d\dot{\gamma}}{d\eta} \left(\frac{E_1}{3E^*}\right)^{1/2} - 1/4 \, \frac{E_1 A_{xx} + E_\nu A_{x\theta}}{E^*} \int_{-1}^{1} U_1 \xi \, d\xi \tag{2.23}$$

$$\dot{m}_\theta = - \frac{1}{2} \frac{E_\nu}{E_1} \frac{d\dot{\gamma}}{d\eta} \left(\frac{E_1}{3E^*}\right)^{1/2} - 1/4 \, \frac{E_2 A_{\theta\theta} + E_\nu A_{x\theta}}{E^*} \int_{-1}^{1} U_2 \xi \, d\xi \tag{2.24}$$

2.3.3 Stress Rate and Strain Rate :

Strain rate and displacement rate relation similar to equation (2.3) is combined with equation (2.22) to get

$$\left.\begin{aligned} \dot{\lambda}_\theta &= \dot{n}_\theta + \frac{1}{2} \int_{-1}^{1} V_2 \, d\xi \\ \dot{\lambda}_x &= \frac{1}{2} \int_{-1}^{1} V_1 \, d\xi - \frac{E_\nu}{E_1} \dot{n}_\theta - \xi \frac{d\dot{\gamma}}{d\eta} \left(\frac{3E^*}{E_1}\right)^{1/2} \end{aligned}\right\} \tag{2.25}$$

Similarly equation (2.2) in terms of rates is substituted in equation (2.19) to get

$$\left.\begin{aligned}\dot{\Sigma}_x &= E_1\,\frac{\dot{\lambda}_x + (E_\nu/E_1)\,\dot{\lambda}_\theta}{E^*} - \frac{E_1\,A_{xx} + E_\nu\,A_{x\theta}}{E^*}\,U_1 \\ \dot{\Sigma}_\theta &= E_2\,\frac{\dot{\lambda}_\theta + (E_\nu/E_2)\,\dot{\lambda}_x}{E^*} - \frac{E_2 A_{\theta\theta} + E_\nu\,A_{x\theta}}{E^*}\,U_2\end{aligned}\right\} \tag{2.26}$$

2.3.4 Equilibrium Equations :

Differentiate equation (2.11) to get its rate form given below.

$$\frac{d\dot{q}}{d\eta} + \dot{n}_\theta = 0 \tag{2.27}$$

Equations (2.22) and (2.23) are combined with equation (2.10) after differentiating it with respect to time to get the following equilibrium equations.

$$\begin{aligned}\frac{d^2\dot{q}}{d\eta^2} - \dot{\gamma} &= \dot{F}_c \\ \frac{d^2\dot{\gamma}}{d\eta^2} + \dot{q} &= \dot{G}_c\end{aligned} \tag{2.28}$$

where

$$\left.\begin{aligned}\dot{F}_c &= \tfrac{1}{2}\,\frac{d}{d\eta}\int_{-1}^{1} V_2\,d\xi \\ \dot{G}_c &= -\frac{3}{2}\left(\frac{E^*}{3E_1}\right)^{\frac{1}{2}}\,\frac{E_1 A_{xx} + E_\nu\,A_{x\theta}}{E^*}\,\frac{d}{d\eta}\int_{-1}^{1} U_1\,\xi\,d\xi\end{aligned}\right\} \tag{2.29}$$

2.4 ELASTICITY AND CREEP EQUATIONS FOR A SPHERICAL SHELL SUBJECTED TO UNIFORM INTERNAL PRESSURE

Sign convention is shown in Figure 2.1. The following equations can be derived in the same manner as described in

sections (2.2) and (2.3). Details of derivation, therefore, have not been included.

Equilibrium equations are

$$\left.\begin{aligned}(L^2 + \frac{E_\nu}{E_1})\,S - 2k^2\,\Theta &= -2k^2\,\frac{\cot\phi}{\alpha}\left(\frac{E_2}{E_1} - 1\right)\\ (L^2 - \frac{E_\nu}{E_1})\,\Theta + 2k^2 S &= 0\end{aligned}\right\}\quad \tau = 0 \qquad (2.30)$$

$$\left.\begin{aligned}(L^2 + \frac{E_\nu}{E_1})\,\dot{S} - 2k^2\,\dot{\Theta} &= 2k^2\,\dot{F}_c\\ (L^2 - \frac{E_\nu}{E_1})\,\dot{\Theta} + 2k^2\,\dot{S} &= 2k^2\,\dot{G}_c\end{aligned}\right\}\quad \tau \geq 0 \qquad (2.31)$$

where

$$\dot{F}_c = -\tfrac{1}{2}\,\mathrm{Cot}\,\phi\int_{-1}^{1}(V_1 - V_2)\,d\xi + \tfrac{1}{2}\frac{d}{d\phi}\int_{-1}^{1} V_2\,d\xi.$$

$$\dot{G}_c = -\frac{3}{2}\left(\frac{E_1}{3E^*}\right)^{1/2}\Bigg[\cot\phi\left\{\int_{-1}^{1}\left(A_{xx} + A_{x\theta}\frac{E_\nu}{E_1}\right)U_1\,\xi\,d\xi - \int_{-1}^{1}\left(A_{\theta\theta}\frac{E_2}{E_1} + A_{x\theta}\frac{E_\nu}{E_1}\right)U_2\,\xi\,d\xi\right\} + \frac{d}{d\phi}\int_{-1}^{1}\left(A_{xx} + A_{x\theta}\frac{E_\nu}{E_1}\right)U_1\,\xi\,d\xi\Bigg] \qquad (2.32)$$

Stress couples and stress resultants are related to rotation Θ and shear force S according to the following equations.

$$n_\theta = \frac{1}{\alpha} - \frac{1}{2k^2}\,\frac{dS}{d\phi}$$

$$n_\phi = \frac{1}{\alpha} - \frac{S}{2k^2}\,\cot\phi \qquad (2.33)$$

$$m_\theta = -\frac{1}{12}\frac{h}{a}\frac{E_2}{E^*}\left(\cot\phi\, H + \frac{E_\nu}{E_2}\frac{dH}{d\phi}\right)$$

$$m_\phi = -\frac{1}{12}\frac{h}{a}\frac{E_1}{E^*}\left(\frac{dH}{d\phi} + \frac{E_\nu}{E_1}\cot\phi\, H\right)$$

Reduced strains and non dimensional stresses are given by

$$\left.\begin{aligned}
\lambda_\theta &= n_\theta - \frac{E_\nu}{E_1} n_\phi - \frac{1}{2}\frac{h}{a}\xi \cot\phi\; H \\
\lambda_\phi &= \frac{E_2}{E_1} n_\phi - \frac{E_\nu}{E_1} n_\theta - \frac{1}{2}\frac{h}{a}\xi \frac{dH}{d\phi} \\
\Sigma_\theta &= \frac{\lambda_\theta + E_\nu/E_2\,\lambda_\phi}{E^*} E_2 \\
\Sigma_\phi &= \frac{\lambda_\phi + E_\nu/E_1\,\lambda_\theta}{E^*} E_1
\end{aligned}\right\} \qquad (2.34)$$

Equations for calculating rate of change of stress resultants and stress couples are

$$\dot{n}_\theta = -\frac{d\dot{S}}{d\phi}\frac{1}{2k^2}$$

$$\dot{n}_\phi = -\dot{S}\,\frac{\cot\phi}{2k^2}$$

$$\dot{m}_\theta = -\frac{1}{12}\frac{h}{a}\frac{E_2}{E^*}\left(\cot\phi\, H + \frac{E_\nu}{E_2}\frac{dH}{d\phi}\right) - \frac{1}{4}\frac{E_2 A_{\theta\theta} + E_\nu A_{x\theta}}{E^*}\int_{-1}^{1} U_2 \xi\, d\xi$$

$$(2.35)$$

$$\dot{m}_\phi = -\frac{1}{12}\frac{h}{a}\frac{E_1}{E^*}\left(\frac{dH}{d\phi} + \frac{E_\nu}{E_1}\cot\phi\, H\right) - \frac{1}{4}\frac{E_1 A_{xx} + E_\nu A_{x\theta}}{E^*}\int_{-1}^{1} U_1 \xi\, d\xi$$

Strain rates and stress rates may be written as

$$\dot{\lambda_\theta} = \dot{n}_\theta - \frac{E_\nu}{E_1}\dot{n}_\phi + \tfrac{1}{2}\int_{-1}^{1} V_2 \, d\xi - \tfrac{1}{2}\frac{h}{a}\xi \cot\phi \, \dot{(H)}$$

$$\dot{\lambda_\phi} = \frac{E_2}{E_1}\dot{n}_\phi - \frac{E_\nu}{E_1}\dot{n}_\theta + \tfrac{1}{2}\int_{-1}^{1} V_1 \, d\xi - \tfrac{1}{2}\frac{h}{a}\xi \frac{d\dot{(H)}}{d\phi}$$

$$\left.\begin{aligned}\dot{\Sigma}_\theta &= E_2 \frac{\dot{\lambda_\theta} + \dot{\lambda_\phi}\, E_\nu/E_2}{E^*} - \frac{E_2 A_{\theta\theta} + E_\nu A_{x\theta}}{E^*} U_2 \\ \dot{\Sigma}_\phi &= E_1 \frac{\dot{\lambda_\phi} + E_\nu/E_1\, \dot{\lambda_\theta}}{E^*} - \frac{E_1 A_{xx} + E_\nu A_{x\theta}}{E^*} U_1\end{aligned}\right\} \tag{2.36}$$

2.5 ELASTICITY AND CREEP EQUATIONS FOR A CIRCULAR CYLINDRICAL SHELL SUBJECTED TO UNIFORM INTERNAL PRESSURE AND AXISYMMETRIC TEMPERATURE DISTRIBUTION

The following stress-elastic strain relations are applicable

$$\left.\begin{aligned}\sigma_x &= E_1 \, \epsilon_{xel} + E_\nu \, \epsilon_{\theta el} - (\alpha_1 E_1 + \alpha_2 E_\nu)\, T \\ \sigma_\theta &= E_2 \, \epsilon_{\theta el} + E_\nu \, \epsilon_{xel} - (\alpha_2 E_2 + \alpha_1 E_\nu)\, T\end{aligned}\right\} \tag{2.37}$$

Equilibrium equations are

$$\left.\begin{aligned}\frac{d^2 q}{d\eta^2} - \gamma &= \frac{dT'}{d\eta} \\ \frac{d^2\gamma}{d\eta^2} + q &= 0\end{aligned}\right\} \quad \tau = 0 \tag{2.38}$$

$$\left.\begin{aligned}\frac{d^2 \dot{q}}{d\eta^2} - \dot{\gamma} &= \dot{F}_c \\ \frac{d^2\dot{\gamma}}{d\eta^2} + \dot{q} &= \dot{G}_c\end{aligned}\right\} \quad \tau \geq 0 \tag{2.39}$$

where

$$\left.\begin{aligned} \dot{F}_c &= \frac{1}{2}\,\frac{d}{d\eta}\int_{-1} V_2{}'\,d\xi \\ \dot{G}_c &= -\frac{3}{2}\left(\frac{E^*}{3E_1}\right)^{1/2}\frac{E_1 A_{xx} + E_\nu A_{x\theta}}{E^*}\,\frac{d}{d\eta}\int_{-1}^{1} U_1'\,\xi\,d\xi \end{aligned}\right\} \qquad (2.40)$$

Taking α equal to unity stress resultants and stress couples may be related to rotation γ and shear force q. Thus,

$$\left.\begin{aligned} n_\theta &= 2 - \frac{dq}{d\eta} \\ m_x &= -\frac{1}{2}\,\frac{d\gamma}{d\eta}\left(\frac{E_1}{3E^*}\right)^{1/2} \\ m_\theta &= m_x\,\frac{E_\nu}{E_1} \end{aligned}\right\} \qquad (2.41)$$

Strains and stresses may be calculated by using equations

$$\left.\begin{aligned} \lambda_x &= \frac{E_2}{E_1} - \text{[redacted]} - \frac{E_\nu}{E_1}\,n_\theta + \frac{\alpha_1}{\alpha_2}\,T' - \xi\,\frac{d\gamma}{d\eta}\left(\frac{3E^*}{E_1}\right)^{1/2} \\ \lambda_\theta &= n_\theta - \frac{E_\nu}{E_1} + T' \\ \Sigma_x &= E_1\,\frac{\lambda_x + E_\nu/E_1\,\lambda_\theta - \left(\frac{\alpha_1}{\alpha_2} + \frac{E_\nu}{E_1}\right)T'}{E^*} \\ \Sigma_\theta &= E_2\,\frac{\lambda_\theta + E_\nu/E_2\,\lambda_x - \left(1 + \frac{\alpha_1}{\alpha_2}\,\frac{E_\nu}{E_2}\right)T'}{E^*} \end{aligned}\right\} \qquad (2.42)$$

Stress resultant rates and stress couple rates are given by

$$\dot{n}_\theta = -\frac{d\dot{q}}{d\eta}$$

$$\dot{m}_x = -\frac{1}{2}\,\frac{d\dot{\gamma}}{d\eta}\left(\frac{E_1}{3E^*}\right)^{1/2} - \frac{1}{4}\,\frac{E_1 A_{xx} + E_\nu A_{x\theta}}{E^*}\int_{-1}^{1} U_1'\,\xi\,d\xi$$

$$\dot{m}_\theta = -\frac{1}{2}\frac{E_\nu}{E_1}\frac{d\dot{\gamma}}{d\eta}\left(\frac{E_1}{3E^*}\right)^{1/2} - \frac{1}{4}\frac{E_2 A_{\theta\theta} + E_\nu A_{x\theta}}{E^*}\int_{-1}^{1} U_2' \xi \, d\xi$$

Strain rates and stress rates are

$$\left.\begin{aligned}
\dot{\lambda}_x &= -\frac{E_\nu}{E_1}\dot{n}_\theta + \frac{1}{2}\int_{-1}^{1} V_1' \, d\xi - \frac{d\dot{\gamma}}{d\eta}\xi\left(\frac{3E^*}{E_1}\right)^{1/2} \\
\dot{\lambda}_\theta &= \dot{n}_\theta + \tfrac{1}{2}\int_{-1}^{1} V_2' \, d\xi \\
\dot{\Sigma}_x &= E_1\frac{\dot{\lambda}_x + E_\nu/E_1\,\dot{\lambda}_\theta}{E^*} - \frac{E_1 A_{xx} + E_\nu A_{x\theta}}{E^*} U_1' \\
\dot{\Sigma}_\theta &= E_2\frac{\dot{\lambda}_\theta + E_\nu/E_2\,\dot{\lambda}_x}{E^*} - \frac{E_2 A_{\theta\theta} + E_\nu A_{x\theta}}{E^*} U_2'
\end{aligned}\right\} \qquad (2.44)$$

2.6 BOUNDARY CONDITIONS

The equations derived in the foregoing sections are to be solved subject to the following Boundary Conditions.

(i) Circular cylindrical shell fixed at an edge (η = constant) :

The necessary conditions are

$$\left.\begin{aligned} \lambda_\theta &= 0 \\ \gamma &= 0 \end{aligned}\right\} \tau = 0 \qquad\qquad (2.45)$$

$$\left.\begin{aligned} \dot{\lambda}_\theta &= 0 \\ \dot{\gamma} &= 0 \end{aligned}\right\} \tau \geqslant 0$$

(ii) Circular cylindrical shell simply supported at an edge (η =constant)

$$\left.\begin{aligned} m_x &= 0 \\ \lambda_\theta &= 0 \end{aligned}\right\} \tau = 0 \qquad\qquad (2.46)$$

$$\left.\begin{aligned} \dot{m}_x &= 0 \\ \dot{\lambda}_\theta &= 0 \end{aligned}\right\} \tau \geqslant 0$$

(iii) Spherical shell fixed at an edge ($\phi = \phi_o$)

$$\left.\begin{array}{l} H = \cdot 0 \\ \lambda_\theta = 0 \end{array}\right\} \tau = 0$$

$$\left.\begin{array}{l} \dot{H} = 0 \\ \dot{\lambda}_\theta = 0 \end{array}\right\} \tau \geqslant 0 \qquad (2.47)$$

Also by symmetry, at apex of the spherical shell

$$\left.\begin{array}{l} S = 0 \\ H = 0 \end{array}\right\} \tau = 0$$

$$\left.\begin{array}{l} \dot{S} = 0 \\ \dot{H} = 0 \end{array}\right\} \tau \geqslant 0 \qquad (2.48)$$

2.7 METHOD OF SOLUTION

Finite difference method has been applied by many researchers to solve elasticity and creep problems of shells of revolution [9,2 An advantage of the method is that no approximations, other than those inherent in the finite difference method, are involved.

To begin with, finite difference method will be applied to the problem of circular cylindrical shell fixed at both the edges. Shell length is divided into n equal parts. Using parabola method of finite difference [20], first and second derivatives may be written for shear force q.

$$\left.\begin{aligned} \frac{dq_n}{d\eta} &= \frac{q_{n+1} - q_{n-1}}{2\,\Delta} \\ \frac{d^2q}{d\eta^2} &= \frac{q_{n-1} - 2q_n + q_{n+1}}{(\Delta)^2} \end{aligned}\right\} \qquad (2.49)$$

Similar expressions may be obtained for other variables also.

Equilibrium equations may, therefore, be written as

For $i = 0, 1, 2, \ldots\ldots. n$ (Fig. 2.1)

$$\left.\begin{aligned} q_{i-1} - 2q_i + q_{i+1} - \Delta^2 \gamma_i &= 0 \\ \gamma_{i-1} - 2\gamma_i + \gamma_{i+1} + \Delta^2 q_i &= 0 \end{aligned}\right\} \quad \tau = 0$$

$$\left.\begin{aligned} \dot{q}_{i-1} - 2\dot{q}_i + \dot{q}_{i+1} - \Delta^2 \dot{\gamma}_i &= \Delta^2 \dot{F}_{ci} \\ \dot{\gamma}_{i-1} - 2\dot{\gamma}_i + \dot{\gamma}_{i+1} + \Delta^2 \dot{q}_i &= \Delta^2 \dot{G}_{ci} \end{aligned}\right\} \quad \tau \geqslant 0 \qquad (2.50)$$

Similarly expressions for boundary conditions (2.45) are obtained (see Appendix A). Incorporating boundary conditions in equation (2.50), 2n simultaneous linear algebraic equations are obtained for each of the cases of elastic deformation ($\tau = 0$) and total ~~creep~~ deformation ($\tau \geqslant 0$). These equations may be represented by

$$[A]\{X\} = \{B\} \qquad \tau = 0 \qquad (2.51)$$

$$[A]\{\dot{X}\} = \{C\} \qquad \tau \geqslant 0 \qquad (2.52)$$

Elements of the matrix $[A]$ and vectors $\{X\}$, $\{B\}$ and $\{C\}$ are given in Appendix A.

Similar matrix equations for simply supported cylindrical shell , clamped spherical shell subjected to uniform pressure and clamped cylindrical shell subjected to non uniform temperature fields may also be obtained (see Appendix A).

It may be observed that coefficient matrix $[A]$ occurs both in the instantaneous equations (2.51) and rate equations (2.52). Vector $\{B\}$ depends on the loading only whereas vector $\{C\}$ depends on current stress state and the load.

Solution of equations (2.51) and (2.52) is the complete solution of the problem. Steps involved in the solution are listed below.

(i) Solution of the initial problem defined by equation (2.51).

(ii) Evaluation of stresses, strains, stress resultants and stress couples by using vector $\{X\}$.

(iii) Substitution of these stresses in expressions for $\dot{F}_c$ and $\dot{G}_c$ to get vector $\{C\}$.

(iv) Solution of the rate problem defined by equation (2.52).

(v) Evaluation of stress rates, strain rates and rates of stress resultants and stress couples by using vector $\{\dot{X}\}$.

(vi) Evaluation of new values of stresses and strains after a small interval $\Delta\tau$ using expressions of the type $\Sigma_{x_{\tau+\Delta\tau}} = \Sigma_{x_\tau} + \dot{\Sigma}_{x_\tau} * \Delta\tau$.

(vii) Steps (iii) to (vi) are repeated till steady state is attained.

2.8 CALCULATION OF REFERENCE STRESS

The main difficulty in creep analysis even under steady loads is the lack of a precise expression for the material behaviour which may be used conveniently. Using curve fitting techniques the data from constant load tests can be approximated by some simple expression.

If structures are subjected to time-dependent loads the data from constant load tensile tests are completely inadequate. To author's ledge ~~no~~ simple method has been found of defining the material creep behaviour under variable loads.

The concept of reference stress as a means of obtaining creep deformation in structures subjected to arbitrary load changes has been found to be promising. The creep behaviour of a structure can directly be linked to strain output from a single strain test conducted at reference stress. Reference stress approach avoids the need for defining material behaviour.

It is possible to choose the unit stress, σ_0, such that the parameters defining the stationary state behaviour of a structure are approximately constant over a range of values of stress index, m . Such a value of unit stress is defined as reference stress, $\overline{\sigma_0}$.

It is unlikely that unit stress, σ_0, first chosen to nondimensionalize the problem will be exactly equal to the reference stress, $\overline{\sigma_0}$. It is desirable, therefore, to use dimensionless solution associated with an arbitrary value of unit stress to obtain parameters associated with reference stress.

Let the creep parameter of interest be strain δ_c at some point in the shell. Let this strain be reduced with respect to ϵ_o and denoted by Δ_c.

$$\frac{d\Delta_c}{d\tau} = \frac{1}{\epsilon_o}\,\frac{d\delta_c}{dt}\cdot\frac{dt}{d\tau}$$

substituting value of $(dt/d\tau)$ obtained from equation (2.14) in the above equation

$$\frac{d\Delta_c}{d\tau} = \frac{1}{P(T)\ B(t)\ \sigma_o^{\ m}}\cdot\frac{d\delta_c}{dt} \tag{2.53}$$

If $$\overline{\sigma}_o = \overline{\alpha}\ \sigma_o\ , \tag{2.54}$$

then the dimensionless rate of parameter, $\frac{d\Delta_c}{d\tau}$ associated with unit stress, σ_o, is related to the dimensionless rate of parameter $\frac{d\overline{\Delta}_c}{d}$ associated with reference stress, $\overline{\sigma}_o$, by the following relation.

$$\dot{\Delta}_c = (\overline{\alpha})^m\ \dot{\overline{\Delta}}_c \tag{2.55}$$

For $m = 0$, $\dot{\Delta}_c = \dot{\overline{\Delta}}_c$

Hence for a particular set of results associated with an arbitrary unit stress, it is possible to calculate the constant $\overline{\alpha}$ using equation (2.55) and hence to determine reference stress using equation (2.54). The minimum number of points required to calculate the reference stress is two. However, when dealing with complex structures, solutions for more than two stress exponent values are desirable.

It may be noted that except in certain cases there is no single reference stress which is applicable for all deformation rates in a structure. In general a particular reference stress is applicable only to a particular deformation parameter.

It may be seen from the definition of dimensionless time (equation 2.14) that its numerical value depends upon unit stress σ_o. It is possible to find a particular value of σ_o so that dimensionless time required to reach stationary stress, τ_{ss}, is independent of stress exponent m. This value of unit stress is called reference stress, associated with stationary state time and is denoted by $\overline{\sigma_{oss}}$ $(=\overline{\alpha}_{ss}\ \sigma_o)$. It may be calculated by using the following relation given in reference 15.

$$\overline{\tau}_{ss} = (\overline{\alpha}_{ss})^{m-1}\ \tau_{ss} \qquad (2.56)$$

For m = 1, $\overline{\tau}_{ss} = \tau_{ss}$

2.9 NUMERICAL METHODS

Matrix equations (2.51) and (2.52) have been solved by Choleski's unsymmetric method[19].

As mentioned already, stresses, strains, and other quantities at any instant, are evaluated by equations of the following type.

$$\left.\begin{aligned} \Sigma_{x_{\tau+\Delta\tau}} &= \Sigma_{x_\tau} + \Delta\tau\,\dot{\Sigma}_{x_\tau} \\ \lambda_{x_{\tau+\Delta\tau}} &= \lambda_{x_\tau} + \Delta\tau\,\dot{\lambda}_{x_\tau} \end{aligned}\right\} \qquad (2.57)$$

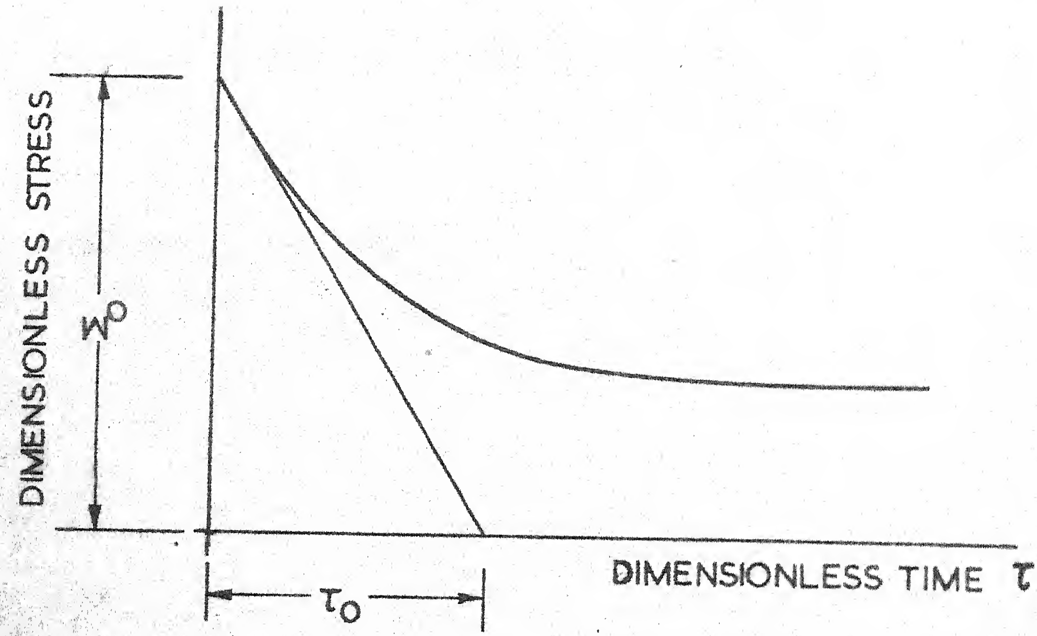

Figure 2.2 : Characteristics of Stress redistribution

Suitable choice of $\Delta\tau$ is very important. Figure 2.2 shows redistribution of stress at some point in the shell starting from its elastic value Σ_o at $\tau = 0$ to stationary state at $\tau = \tau_{ss}$. At time $\tau = 0$, Σ_o and $\dot{\Sigma}_o$ are precisely known so that the intercept of line $\Sigma = \Sigma_o + \dot{\Sigma}_o\tau$ with the abscissa defines the point $\tau = \tau_o$. It is not desirable to choose $\Delta\tau \geqslant \Sigma_o/\dot{\Sigma}_o$ $(= \tau_o)$ because this would lead to absurdly hasty redistribution. A better choice is $\Delta\tau = \sigma_o/f$; where $f \gg 1$. In general

$$\Delta\tau = \frac{1}{f}\left|\frac{\Sigma}{\dot{\Sigma}}\right|, \qquad f \gg 1 \tag{2.58}$$

An advantage of scheme given by equation (2.58) is that $\Delta\tau$ chosen in this manner by using local values of Σ and $\dot{\Sigma}$ can be used throughout the computations. As time increases, rate $\dot{\Sigma}$ decreases and incremental time $\Delta\tau$ increases. A more general formula will be

$$= \frac{1}{f} \text{ minimum of} \left|\left(\frac{\Sigma}{\dot{\Sigma}}, \frac{m_x}{\dot{m}_x}, \frac{\lambda_x}{\dot{\lambda}_x}, \frac{n_x}{\dot{n}_x}, \ldots \text{etc.}\right)\right| \tag{2.59}$$

It may be remarked that such a scheme will require excessive computational effort. A better approach is to use equation (2.58) with a conservative value of factor f.

When using equations (2.58) or (2.59) care should be taken to avoid those points in the structure where stresses (or strain etc.) are very low. Very low stress may lead to incremental time approximately equal to zero.

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$$\Delta\tau = \frac{1}{f} \left| \frac{\Sigma}{\dot{\Sigma}} \right| , \qquad f \gg 1 \tag{2.58}$$

An advantage of scheme given by equation (2.58) is that $\Delta\tau$ chosen in this manner by using local values of Σ and $\dot{\Sigma}$ can be used throughout the computations. As time increases, rate $\dot{\Sigma}$ decreases and incremental time $\Delta\tau$ increases. A more general formula will be

$$= \frac{1}{f} * \text{minimum of} \left| \left(\frac{\Sigma}{\dot{\Sigma}} , \frac{m_x}{\dot{m}_x} , \frac{\lambda_x}{\dot{\lambda}_x} , \frac{n_x}{\dot{n}_x} , \ldots \text{etc.} \right) \right| \tag{2.59}$$

It may be remarked that such a scheme will require excessive computational effort. A better approach is to use equation (2.58) with a conservative value of factor f.

When using equations (2.58) or (2.59) care should be taken to avoid those points in the structure where stresses (or strain etc.) are very low. Very low stress may lead to incremental time approximately equal to zero.

In addition to a suitable choice of incremental time, $\Delta\tau$, it is necessary to decide when to stop computations. Stationary state may be defined based on one of the following criteria.

(i) Strain rate is approximately constant

(ii) Strain rate and creep strain rate are approximately equal

(iii) Stress rate is approximately zero.

In order to minimize computations without losing results of practical interest, it is suggested that computations be stopped when stress rate at a point, where strain rate is maximum, approaches a low value. This is necessary because of the asymptotic nature of stress variation with time as steady state is approached. It is suggested that when stress rate at such a point is less than 0.01, the computations may be stopped.

RESULTS AND DISCUSSION

Results were computed for clamped circular cylindrical shell, simply supported cylindrical shell and clamped spherical shell subjected to a uniform internal pressure. Values used for the creep law exponent were 3, 5, 7, 9. Spherical shell problem was solved for an isotropic material whereas cylindrical shell problem was solved for the following values of the coefficients of anisotropy.

$$0.6 \leq A_{\theta\theta} \leq 1.2 \; ; \; -0.3 \geq A_{x\theta} \geq -0.6$$

This range of properties covers a wide range of materials. A problem of clamped circular cylindrical shell made of Zircaloy-2 and subjected to a uniform pressure and axisymmetric linear temperature distribution was also solved.

The problems of cylindrical shell subjected to uniform pressure were solved for non-dimensional length of 80. The problems of spherical shell were solved for subtended angle ϕ_0 equal to 30 degrees and ratio of radius to thickness equal to 50. The non-dimensional length of cylindrical shell subjected to linear temperature field was taken equal to 40.

Unless otherwise stated, for all the problems E_1 and E_2 are equal. Ratio of E_ν to E_1 is 0.3.

3.1 SHELLS SUBJECTED TO UNIFORM PRESSURE

The main features of behaviour of shells are shown in Figures 3.1 through 3.8 for clamped cylindrical shell, Figures 3.9 through 3.13 for simply supported cylindrical shell, Figures 3.14 through 3.20 for spherical shells. In all the cases unit stress is $\frac{pa}{2h}$. The results have been shown for nodes

starting from the shell edge to its middle. It is observed that

(i) Axial stress at clamped edge of cylindrical shell is diminished from its initial value at $\tau = 0$ to a stationary value at $\tau = \tau_{ss}$ by the process of creep (Figure 3.1). The higher the value of m, the creep exponent, the higher is the strain rate and, therefore, the quicker is the stress redistribution process. Similar stress redistribution is observed in the case of simply supported cylindrical shell (Figure 3.10) and clamped spherical shell (Figure 3.16).

(ii) The effect of clamping in introducing stress concentration is experienced farther from the edge as compared to elastic case (Figures 3.3 and 3.4).

Although creep process is powerful in diminishing peak stresses, the stationary state values of peak stresses are relatively insenitive to the values of creep law exponent m as is the distribution along the shell (Figures 3.4, 3.9 and 3.15). It is interesting to note that some skeletal points are present in shells. Location of skeletal points is given in Table 3.1. The points where stress is independent of creep law exponent have also been included in the same table.

The effect of redistribution of stresses on rupture times are tempting to speculation. If one were to use some sort of cumulative damage law in conjunction with rupture data from steady load tests, rupture times would be very much reduced from the values obtained by using stationary state stress.

TABLE 3.1

Location of skeletal points along shell meridian

Type of shell	Material	Position of skeletal point	Comments	
1. Clamped cylindrical	Isotropic	$\eta = 3$	a	Fig. 3.3
		$\eta = 0.7$	b	Fig. 3.4
	Anisotropic[c]	$\eta = 0.3$	b	Fig. 3.4
		$\eta = 0.7$	b	Fig. 3.4
2. Simply supported cylindrical	Isotropic	$\eta = 1.5$	b	Fig. 3.9
	Anisotropic[c]	$\eta = 1.5$	b	Fig. 3.9
3. Clamped spherical	Isotropic	$\phi = 75°$	a	Fig.3.14
		$\phi = 26°$	b	Fig.3.15

a Stress at stationary state is same as elastic value

b Stress changes with time but its stationary state value is independent of stress exponent

c $A_{\theta\theta} = 0.6$, $A_{x\theta} = -0.3$

(iii) Distribution of stress across shell wall is shown in Figures 3.2, 3.11 and 3.17 for clamped cylindrical, simply supported cylindrical and clamped spherical cases respectively. As expected, the distribution becomes nonlinear, unlike the linear distribution in elastic case. Higher the value of stress exponent, greater is the deviation from elastic solution. Again it may be observed that there exist skeletal points at

(a) $\xi_f = -0.46$ for clamped cylindrical shell

(b) $\xi_f = +0.05$ for simply supported cylindrical shell

(c) $\xi_f = -0.90$ for clamped spherical shell.

(iv) With reference to Figures 3.5, 3.7, 3.13, 3.19 it will be seen that rate of increase of strain is maximum at time $\tau = 0$. As the shell creeps, the stresses decrease and, therefore, strain rates decrease until they attain a minimum value at stationary state.

It may be remarked that strains estimated by relation

$$\epsilon = \epsilon_{el} + \dot{\epsilon}_{ss} * t$$

will lead to reasonably accurate results for $m \leq 5$.

Distribution of circumferential strain in clamped cylindrical shell and spherical shell at different times $0 \leq \tau \leq \tau_{ss}$ is shown in Figures 3.6 and 3.18 respectively.

(v) Variation of stationary state strain rates along shell meridian is similar to stress distribution at stationary state (Figures 3.8, 3.12 and 3.20). Strain rate is maximum at points where stress is maximum. Thus in clamped cylindrical and spherical shells axial strain has maximum value at the edge. In simply supported cylindrical shells circumferential strain rate is maximum at the middle of length.

(vi) Real time values can be calculated from τ by using equation (2.15). Time required to reach steady state may vary from a few hours to a few hundred hours (Table 3.2). It may be observed that a vessel operating for two shifts in a day will never attain a stationary state. Thus stationary state analysis as advanced by many authors[5,6,7,8] may lead to non-conservative results.

TABLE 3.2

Time Required to Attain Steady State (a)

Material		Type of shell	Temp. (°c)	m	σ_o (kg/cm^2)	τ_{ss}	t_{ss} (hrs.)
1.	b	Clamped cylindrical	800	3	400	5.58	31
		Simply supported cylindrical	800	3	400	1.35	8
		Spherical	800	3	400	3.45	19
2.	c	Clamped cylindrical	650	5	500	1.25	75
		Simply supported cylindrical	650	5	500	0.95	58
		Spherical	650	5	500	3.11	190

(a) All the properties have been taken from reference 16. The creep law used in this reference is

$$\frac{d\epsilon_c}{dt} = 10^{-7} \left(\frac{\sigma}{\sigma_{c7}}\right)^m$$

b 18 Cr 8 Ni 0.45 Si 0.4 Mn 0.1 C stainless steel
$E = 1.4 \times 10^6$ kg/cm^2 , $\sigma_{c7} = 50$ kg/cm^2

c 25 Cr 20 Ni 1.5 Si 1Mn 0.1 C stainless steel
$E = 1.4 \times 10^6$ kg/cm^2 ; $\sigma_{c7} = 350$ kg/cm^2

3.2 EFFECT OF ANISOTROPY

$A_{\theta\theta}$ and $A_{x\theta}$ are two independent constants used to describe the anisotropy of an orthotropic material. As explained in reference 17, if $A_{\theta\theta} < A_{xx}$, then $A_{x\theta} > -0.5$.

Similarly if $A_{\theta\theta} > A_{xx}$, then $A_{x\theta} < -0.5$. Keeping this in view the following four cases were studied for simply supported cylindrical shell and clamped cylindrical shell. Stress exponent in each case was taken as 3.

(a) $A_{\theta\theta} = 0.6$ $\quad A_{x\theta} = -0.3$

(b) $A_{\theta\theta} = 0.8$ $\quad A_{x\theta} = -0.4$

(c) $A_{\theta\theta} = 1.0$ $\quad A_{x\theta} = -0.5$ (Isotropic Material)

(d) $A_{\theta\theta} = 1.2$ $\quad A_{x\theta} = -0.6$

The results have been plotted in Figures 3.21 through 3.26. The following observations are made.

(i) Strain rates at stationary state are minimum for case (a) and maximum for case (d) (Figures 3.21 and 3.22). Referring to equations (2.17 and 2.18) it is seen that decrease in values of $A_{\theta\theta}$ and $A_{x\theta}$ results in decrease of creep strain rate. Comparing any two of examples under study it will be seen that an increase in the value of $A_{x\theta}$ and decrease in the value of $A_{\theta\theta}$ reduce strain rate. Thus it may be concluded that value of $A_{\theta\theta}$ is more important.

(ii) As has been already remarked, higher the creep rate, smaller is the value of maximum stress at stationary state (Figures 3.23 through 3.26).

(iii) Higher the creep rate, quicker is the stress redistribution. Therefore, time to reach steady state is minimum for case (d) and maximum for case (a). (Figures 3.23 and 3.24).

(iv) Variation of circumferential strain rate in simply supported cylindrical shell is shown in Figure 3.27. Calculation of strain by superposition of elastic strain and stationary values does not involve significant error.

Spherical shell problem has not been solved for anisotropic material because degree of anisotropy of a spherical shell may vary along meridian and the shell tends to be isotropic at its apex when the shell properties are axisymmetric.

Since anisotropy coefficients depend on texture of material, it may be possible to choose a fabrication process which develops a texture that gives minimum creep strain rate.

3.3 THERMAL STRESSES

A clamped cylindrical shell subjected to a uniform internal pressure and an axisymmetric temperature distribution varying linearly along the length of the shell was analysed. The results are shown in Figures 3.28 and 3.29. The shell is made of zircaloy-2 which follows a law given by equation (2.1) with

$$P(T) = \phi' \times 26 \times 10^{-13} \quad e^{\frac{-8390}{T}} \qquad \text{(Reference 17)}$$

$$B(t) = 1$$

Stationary state axial creep rate at the hot end was 5.07×10^{-7} per hour as compared to 2.71×10^{-6} per hour at the cold end when normalized flux was taken as 2.7 and unit stress was 7500 psi. Maximum circumferential creep strain rate which occured at the middle of the shell was 0.69×10^{-7} per hour.

Creep caused by external loads leads to redistribution of stresses, drop in stresses at discontinuities being made up

by increase in stress at other points. Since the thermal stresses are internally in equilibrium, they decrease throughout the whole volume of structure and tend, gradually, to disappear completely. To illustrate this, clamped cylindrical shell subjected to uniform pressure and uniform temperature was analysed for temperature rise of 280°c and compared with analysis for temperature rise of 30°c. In both the cases stresses at stationary state were the same. (Figure 3.30).

3.4 REFERENCE STRESS

Figures 3.31 and 3.32 show the dependence of stationary state strain rate and time τ_{ss} on stress exponent m. The form of numerical results depends on the magnitude of unit stress, σ_o $(= \alpha \frac{pa}{2h})$. As may be seen, there are particular values of unit stress at which the strain rates and time τ_{ss} are approximately constant. Such a value of unit stress is the reference stress for the particular parameter. Following points are observed.

(i) Each of the parameters, that is, axial strain rate at a point, circumferential strain rate at a point and time τ_{ss} has a different value of reference stress associated with it.

(ii) Computed results for strain rates behave almost according to equation (2.55), the deviation being within $\pm$ 4 per cent(Figure 3.31). Therefore, it is possible to calculate $\bar{\alpha}$, the parameter defining reference stress, accurately. The accuracy can be attributed to two factors. Firstly, very small change in the value of α is

magnified m times when plotting. Secondly, since the dimensionless strain rate is convergent to a fixed value, its numerical evaluation is relatively isensitive to stationary state criterion used.

(iii) Dimensionless times τ_{ss} do not conform to equation (2.56) to the same degree of accuracy as strain rates do to equation (2.55). The numerical methods used make it difficult to determine the stationary state time accurately. The results depend on stationary state criterion used and on some programming parameters. Scatter was found to be within + 15 and -4 percent.

(iv) It may be seen from Figure 3.31 that reference stress for calculating the stationary state strain rate at a point in shell depends on anisotropy coefficients though it is independent of stress exponent m. Thus use of reference stress analysis to predict minimum strain rate by conducting one uniaxial creep test at reference stress is possible only when anisotropy coefficients are known. This is not a serious limitation since anisotropy coefficients can be related to texture of material[17].

(v) Minimum strain rate obtained by reference stress analysis may be used to get strain at any time τ according to equation

$$\lambda = \lambda_{el} + \dot{\lambda}\,\tau$$

Strain calculated in this way constitutes a lower bound. The measured strain will be always greater than the predicted value using this method because of additional work done as stresses redistribute from elastic to stationary state

values. The difference, however, may not be much if structure is under creep for a sufficiently long time.

In addition, however, since the effect of stress redistribution is included in the original dimensionless solutions it is easy to obtain an approximate upper bound to deformation of shell by using an upper estimate of the stress exponent(Figure 3.33).

3.5 NOTES ON COMPUTATIONS

The computations involve solution of matrix equations of the type

$$[A]\{X\} = \{B\}$$

Unsymmetrical method of Cheloski was used. Step lengths of 0.1 and 0.5° were found sufficient for cylindrical and spherical shells respectively. The elasticity solutions were compared with analytical solutions given in reference 18. Maximum error in stress values was -0.2 per cent. Stress distribution in a shell subjected to uniform pressure should be symmetrical about the middle of length. Maximum deviation from symmetry of stress distribution was negligible for elastic solution and 1 percent for stationary state solution. Also by using Richardson extrapolation technique [20] with step size equal to one half of the step size suggested earlier, a maximum error of -0.5 per cent was estimated.

Simpson formula with $(m + 2)$ sections in half thickness was used to evaluate integrals like $\dot{F}_c$ and $\dot{G}_c$.

Errors estimated by Richardson technique[19] were found negligible.

Incremental time was calculated according to equation

$$\Delta\tau = \frac{1}{f}\left|\left(\frac{\sum x}{\sum \dot{x}}\right)\right| \text{ for clamped shells}$$

$$\Delta\tau = \frac{1}{f}\left|\left(\frac{\sum \theta}{\sum \dot{\theta}}\right)\right| \text{ for simply supported shells}$$

Many values of factor f were tried. It was found that solutions no longer changed with $f \gg 10$.

All the problems were solved on IBM 7044 computer. One problem requires 3 to 15 minutes of computer time depending on the value of creep law exponent. Higher the value of exponent, more is the time required.

CHAPTER IV

CONCLUSIONS

In the present work a method of creep analysis of cylindrical and spherical shells has been developed. The method can be applied to determine stress redistribution and strain accumulation in shells made of orthotropic materials. Important conclusions from this work are listed below.

(i) Creep leads to redistribution of stresses. High stresses near discontinuities are reduced while stresses at other points of the structure increase. Since the effect of stress concentration in shells is significantly less in creep than in elastic case, it may be inferred that membrane theory gives more accurate results in the former case.

(ii) Stress redistribution is quicker and strain rates are higher for higher values of creep law exponent.

(iii) When calculating rupture life of shell, stress redistribution should be taken into consideration. Estimates based on stationary state values may lead to non conservative results.

(iv) Strain rates are maximum at the time of loading and they decrease to a minimum value at stationary state.

(v) Time taken by a shell to reach stationary state may vary from a few hours to a few hundred hours. Shells which are not in operation for long durations may never attain stationary state.

(vi) A reduction in the values of anisotropy coefficients $A_{\theta\theta}$ and $A_{x\theta}$ results in a decrease in stationary state strain rates and an increase in time required to reach stationary state.

(vii) A decrease in the value of anisotropy coefficient $A_{\theta\theta}$ and an increase in the value of coefficient $A_{x\theta}$ within some bounds may also reduce stationary state strain rates.

It may be possible to select fabrication processes to develop a texture which gives minimum stationary state strain rates.

(viii) Thermal stresses diminish throughout the structure. Stationary state stress distribution is independent of the temperature at which the shell operates.

(ix) Reference stresses corresponding to stationary state strain rate and time to stationary state have been calculated for $3 \leqslant m \leqslant 9$. With knowledge of anisotropy coefficients one uniaxial creep test conducted at the reference stress gives stationary state deformation at a point of interest. The necessity of evaluating the stress exponent can thus be obviated.

(x) Strain calculated by reference stress method is the lower bound to the stationary state strain. Its value calculated by direct creep analysis (using an upper estimate of creep law exponent) constitutes an approximate upper bound to the stationary state strain.

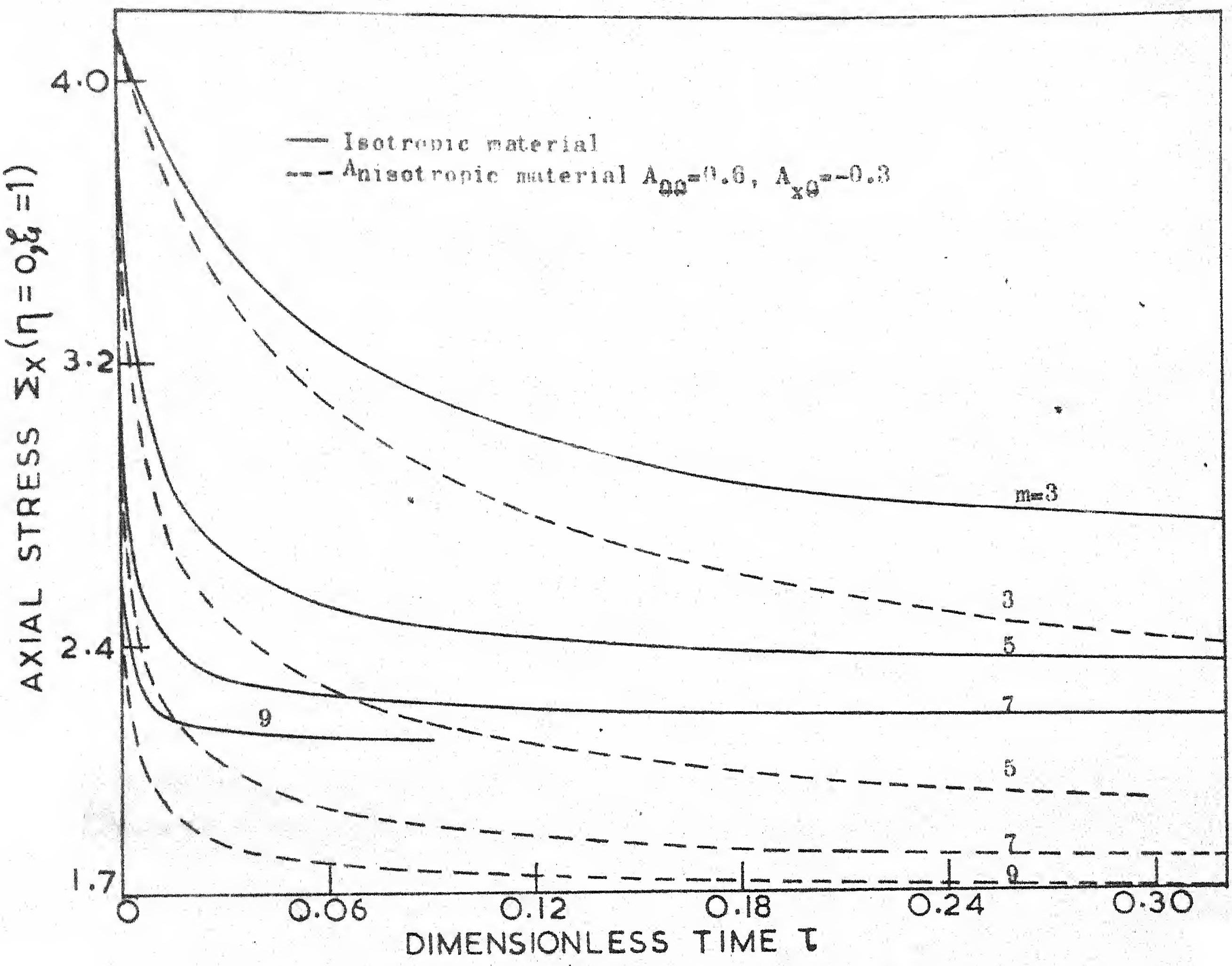


Figure 3.1 : Variation of maximum surface stress in clamped cylindrical shell with time:influence of stress exponent m.

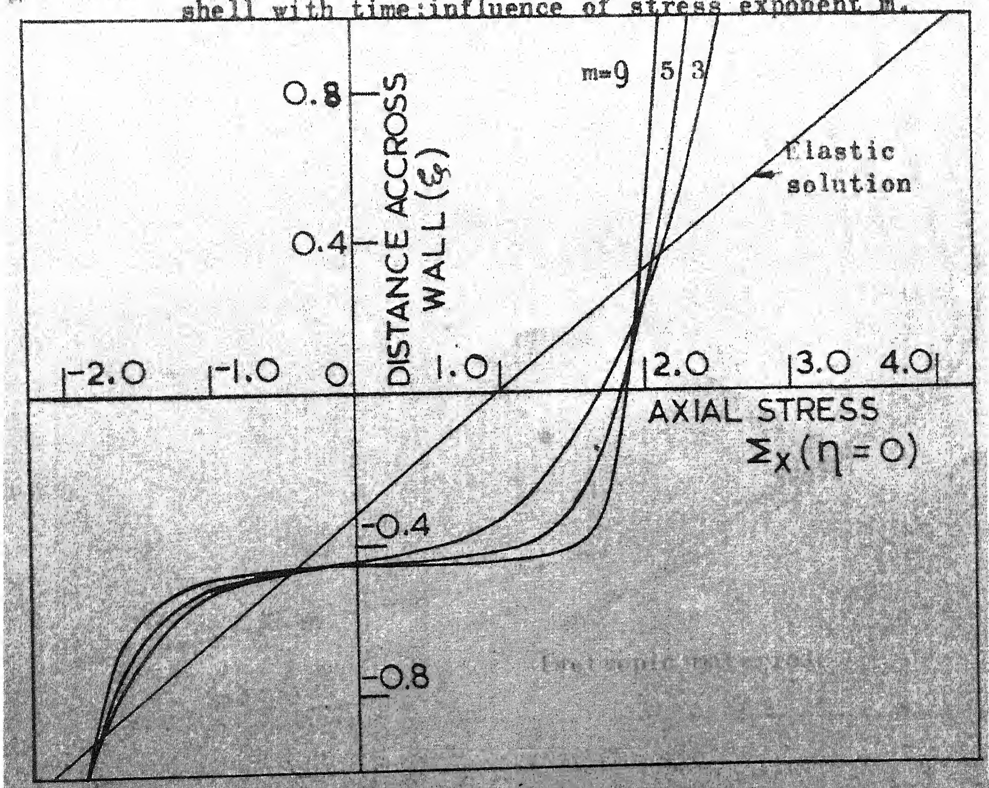


Figure 3.2 : Distribution of axial stress in clamped cylindrical shell at its root. Values at [illegible]

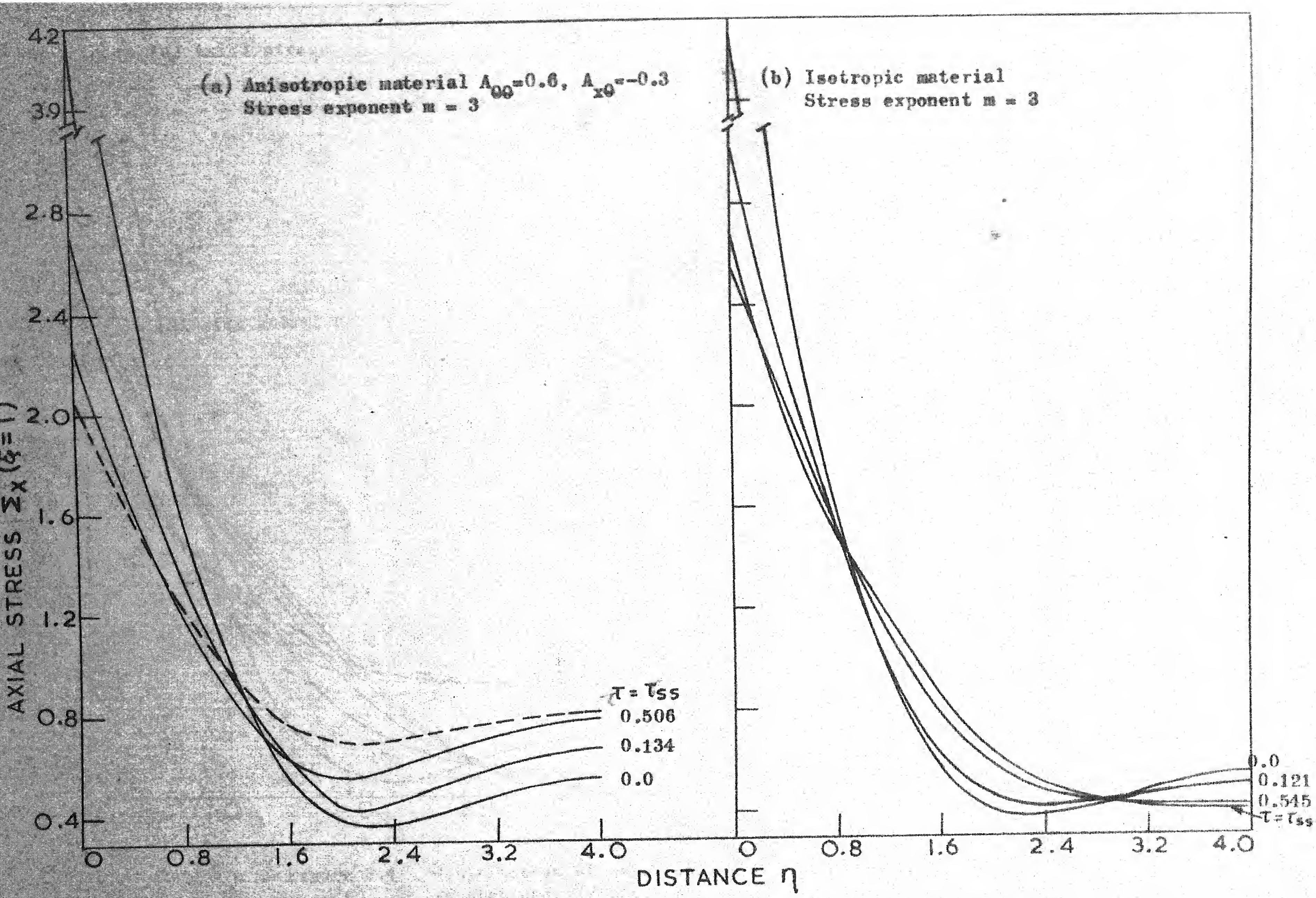


Figure 3.3 : Variation of axial stress in clamped cylindrical shell with time

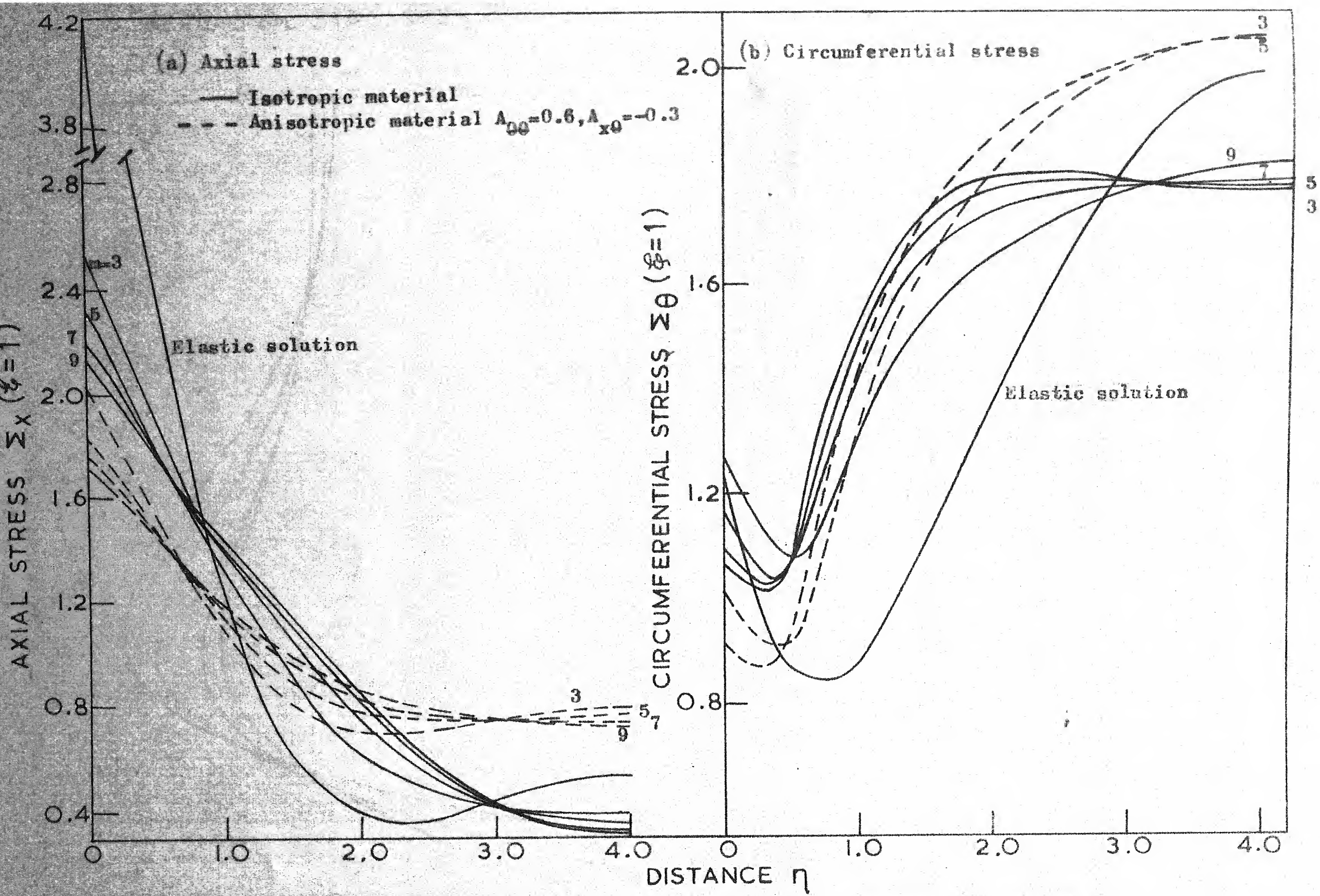


Figure 3.4 : Distribution of axial stress in clamped cylindrical shell at stationary state : influence of stress exponent m.

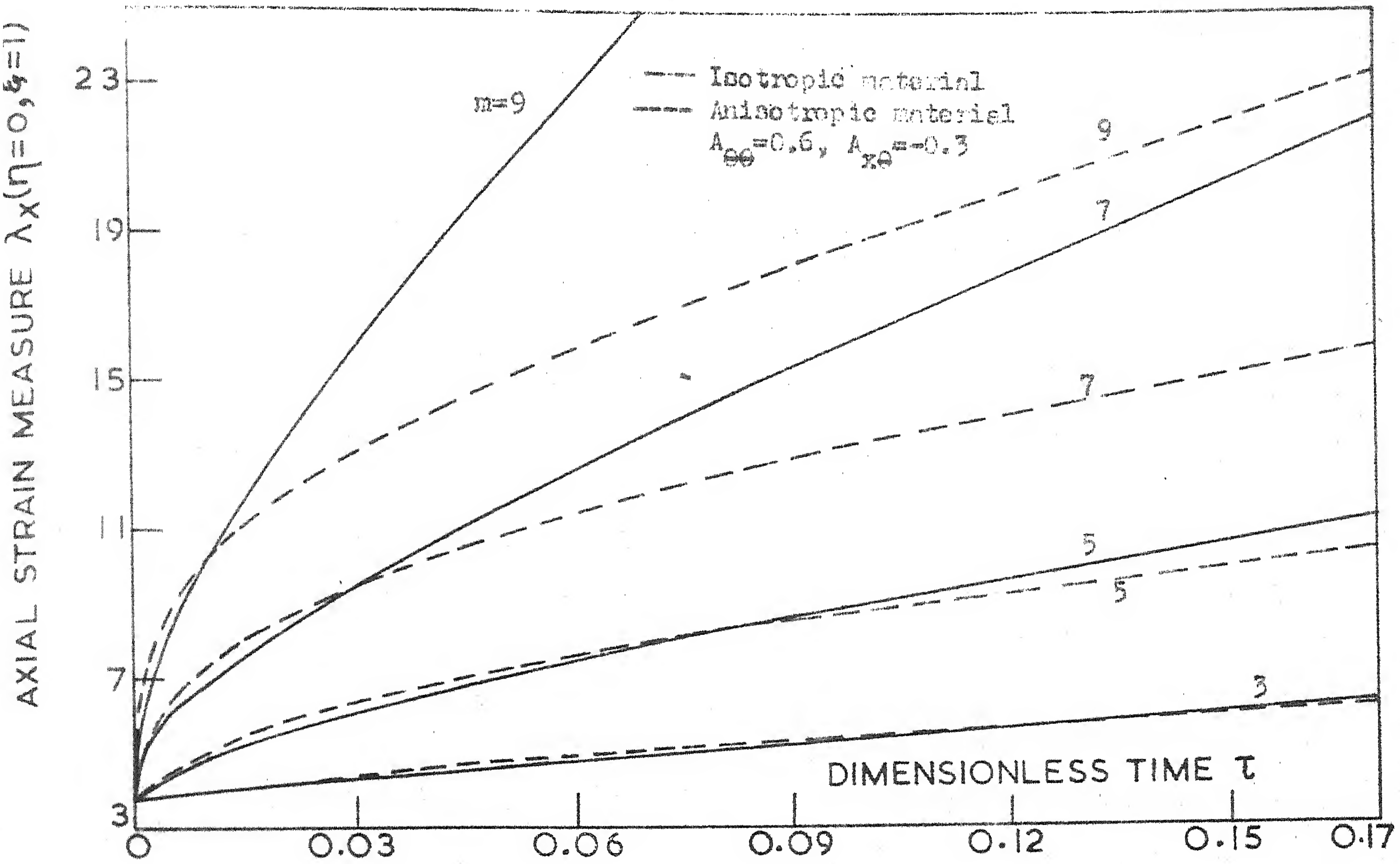


Figure 3.5 : Variation of axial strain at clamped cylindrical shell root with time : influence of stress exponent m.

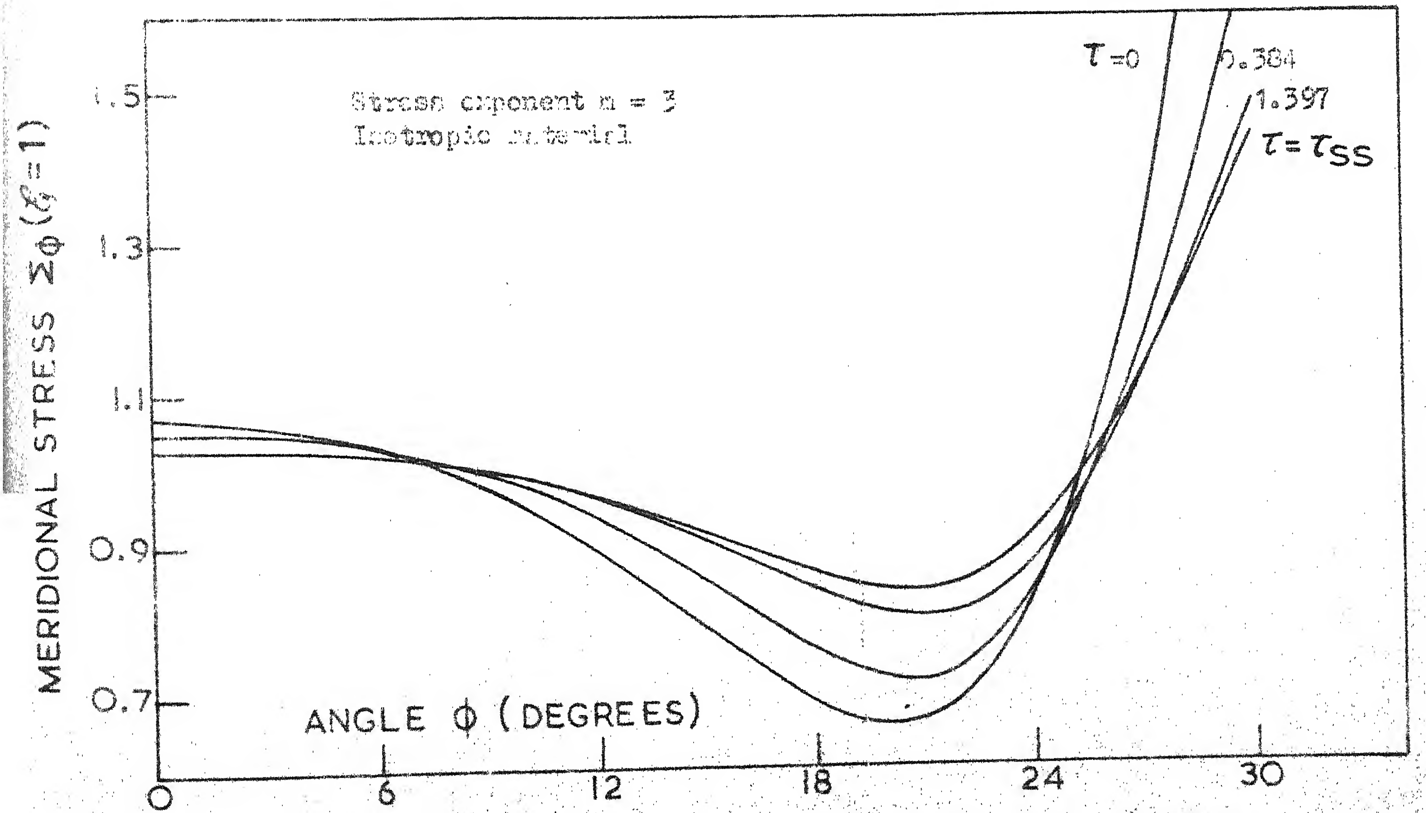


Figure 3.1[illegible] : Variation of meridional stress in clamped spherical shell with time

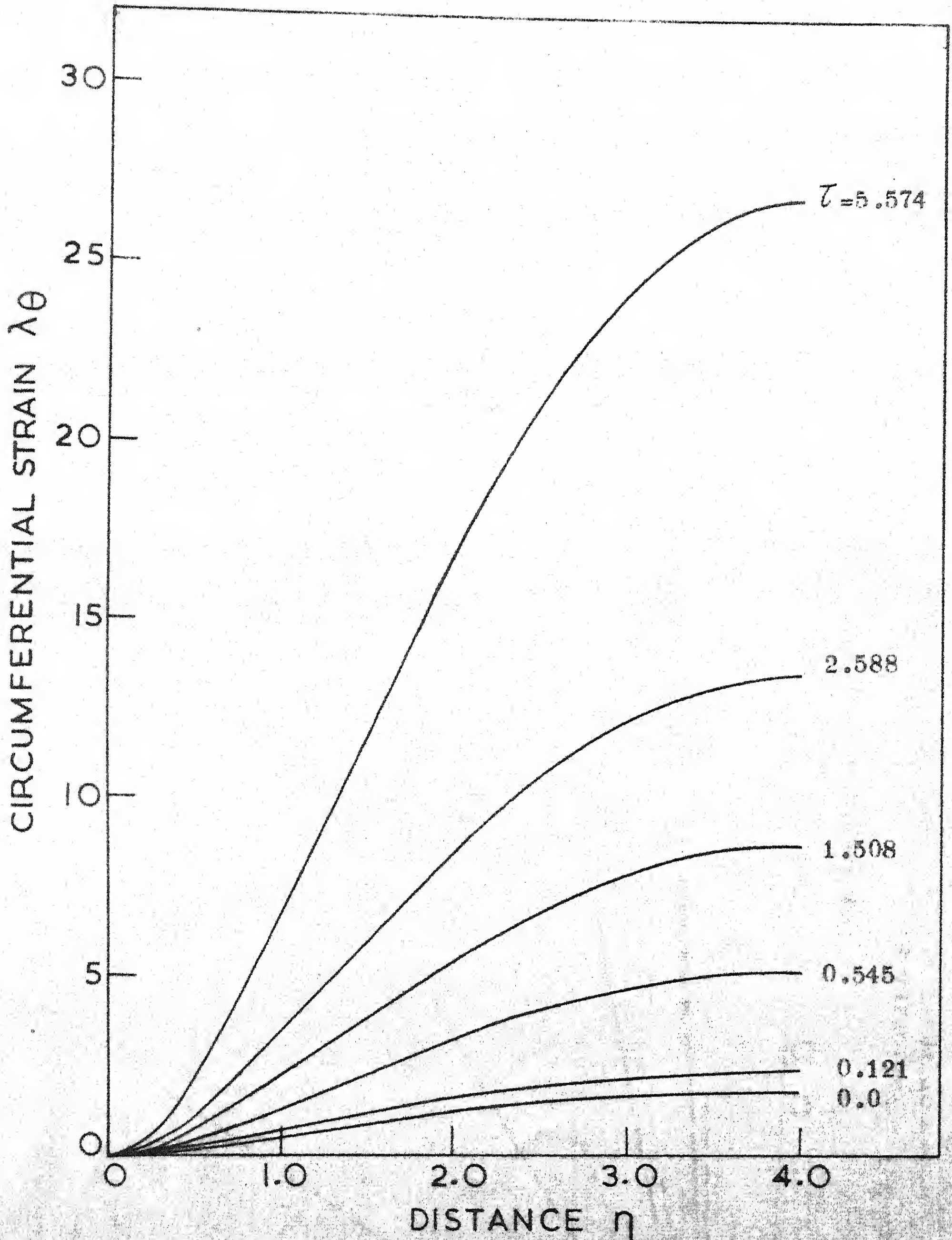


Figure 3.6 : Variation of Circumferential strain in clamped cylindrical shell with time : influence of stress exponent m. Isotropic material.

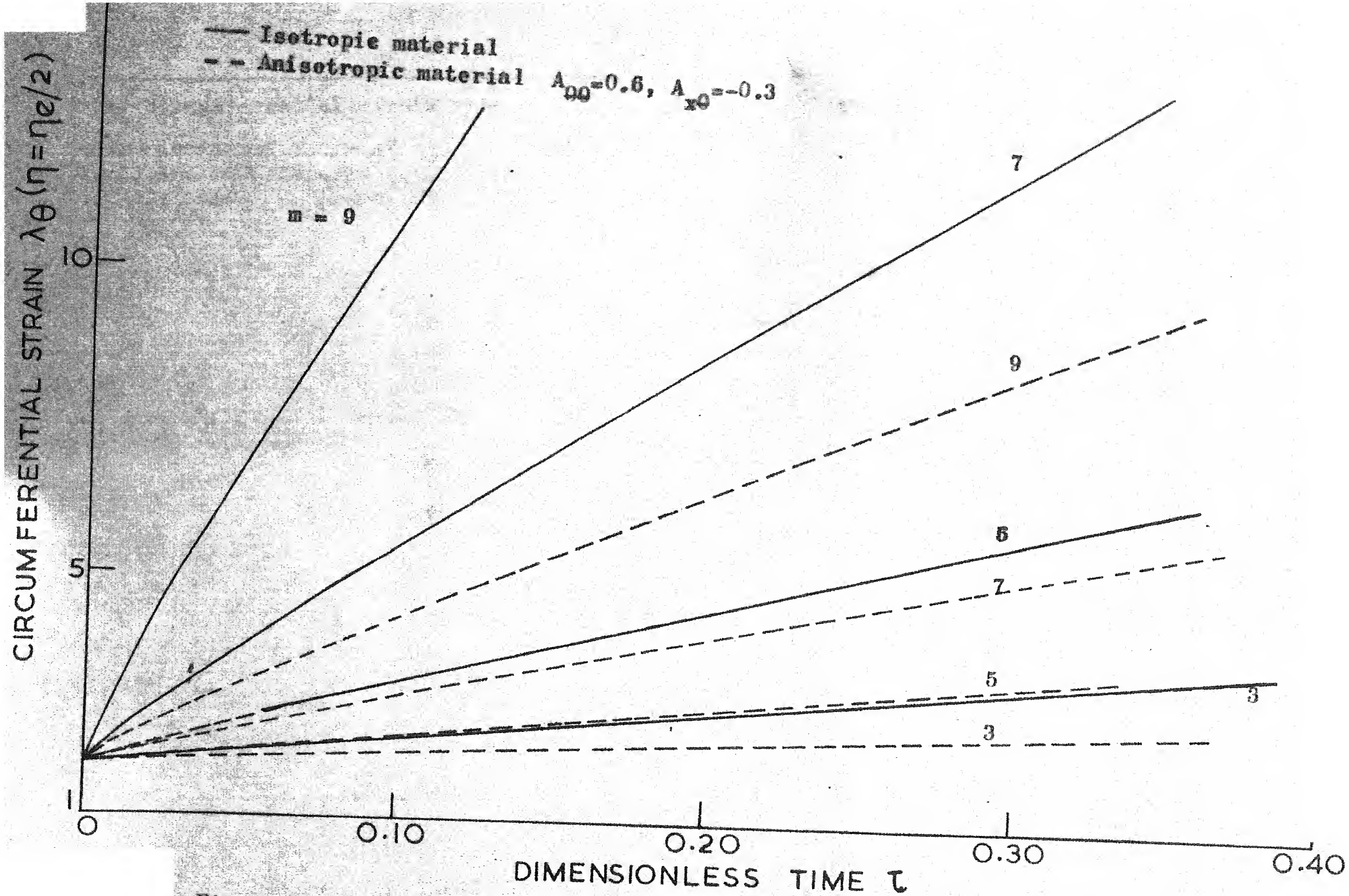


Figure 3.7 : Variation of circumferential strain in middle of clamped cylindrical shell with time : influence of stress exponent m

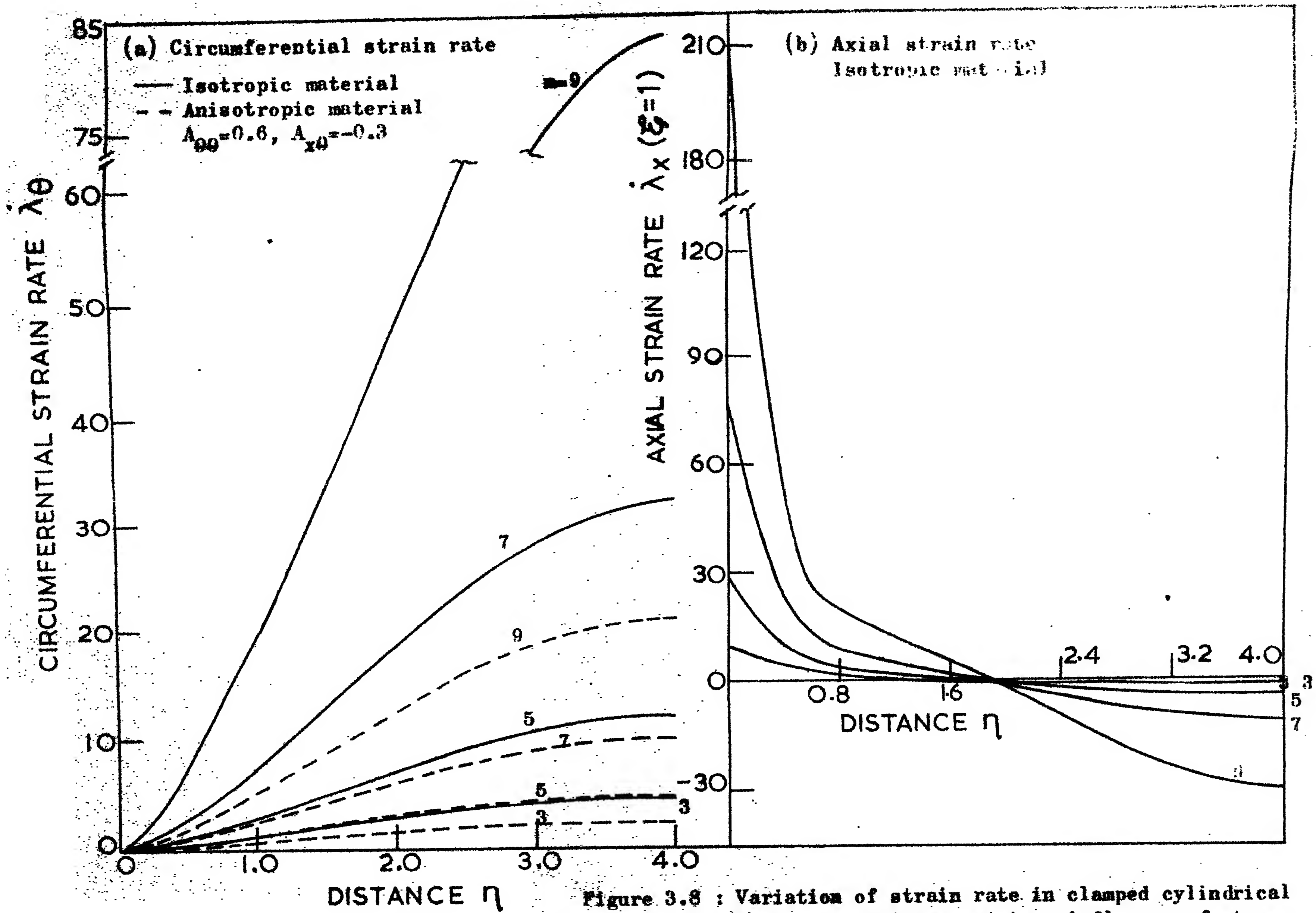


Figure 3.8 : Variation of strain rate in clamped cylindrical shell at stationary state : influence of stress exponent m.

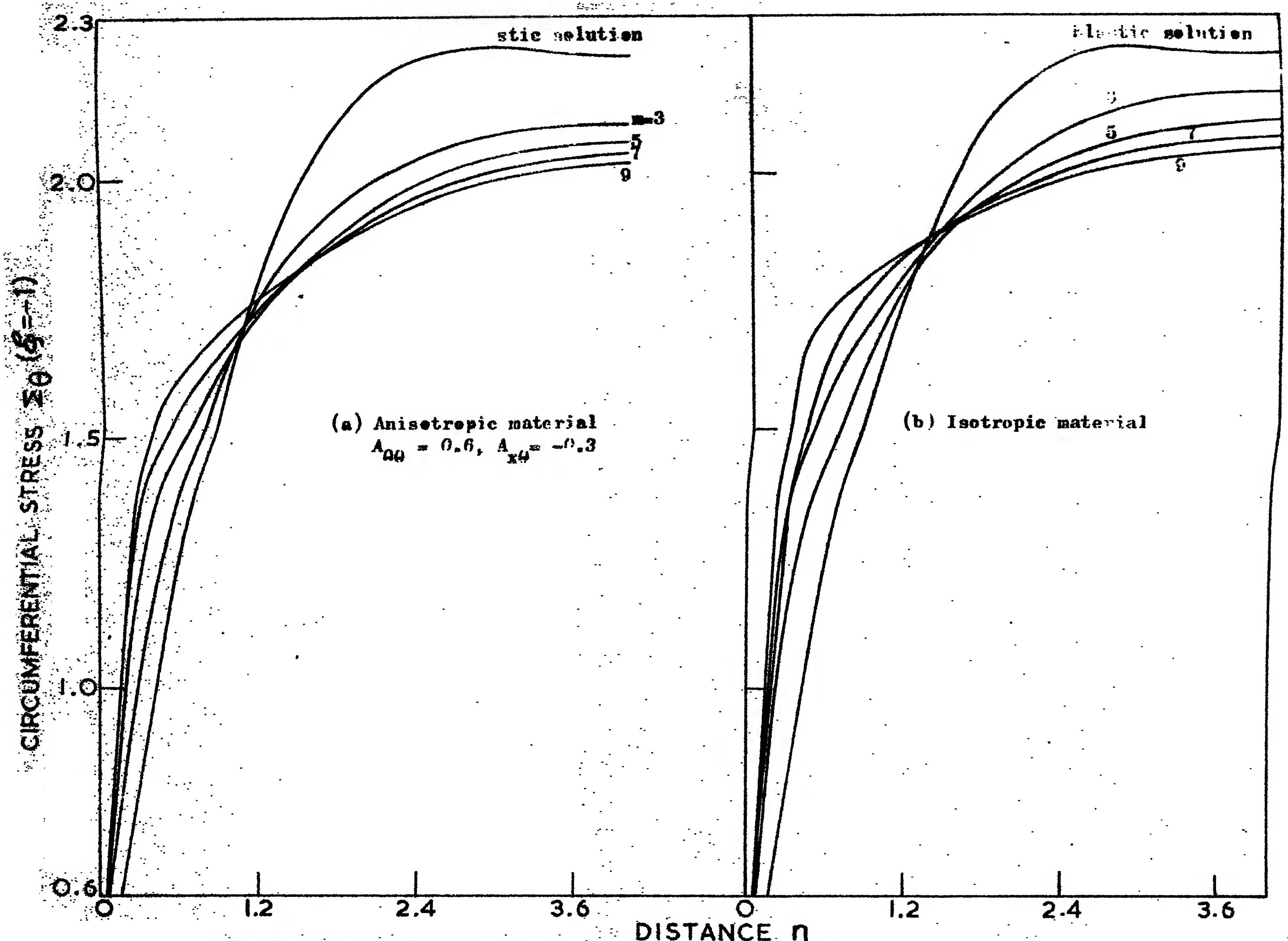


Figure 3.9 : Distribution of circumferential stress in simply supported cylindrical shell at stationary state : influence of stress exponent m.

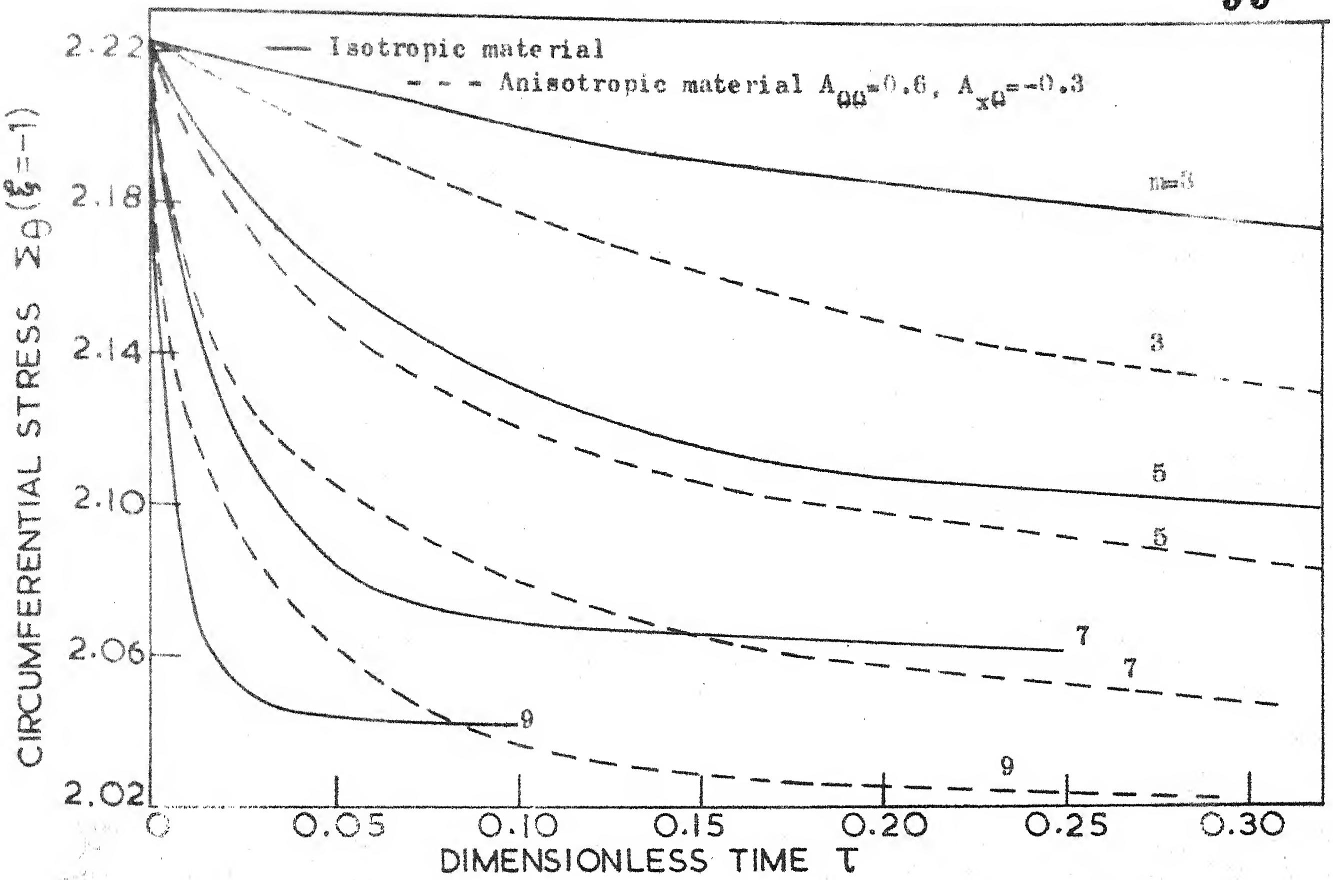


Figure 3.10 : Variation of circumferential stress at middle of simply supported cylindrical shell : influence of stress exponent m .

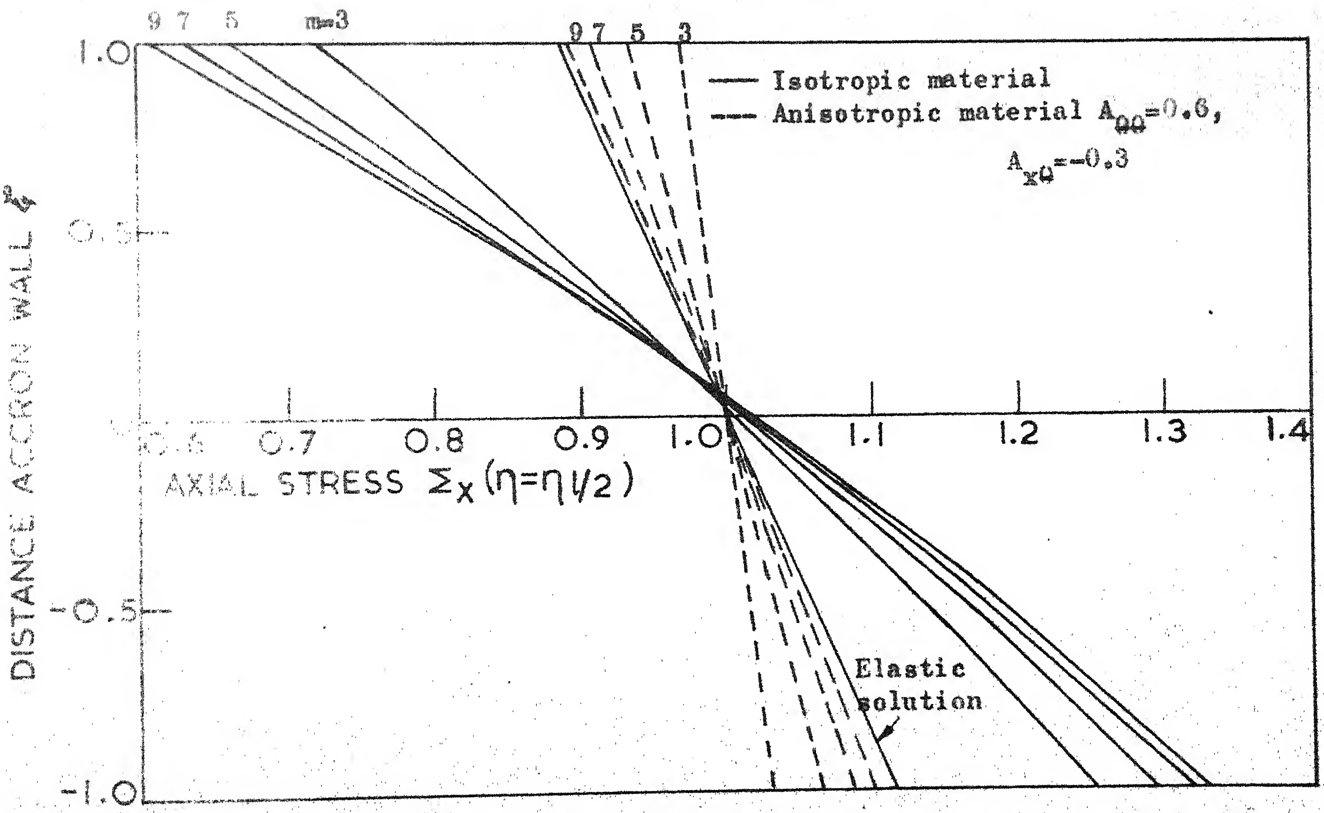


Figure 3.11 : Distribution of axial stress at middle of simply supported cylindrical shell at stationary state : influence of stress exponent m .

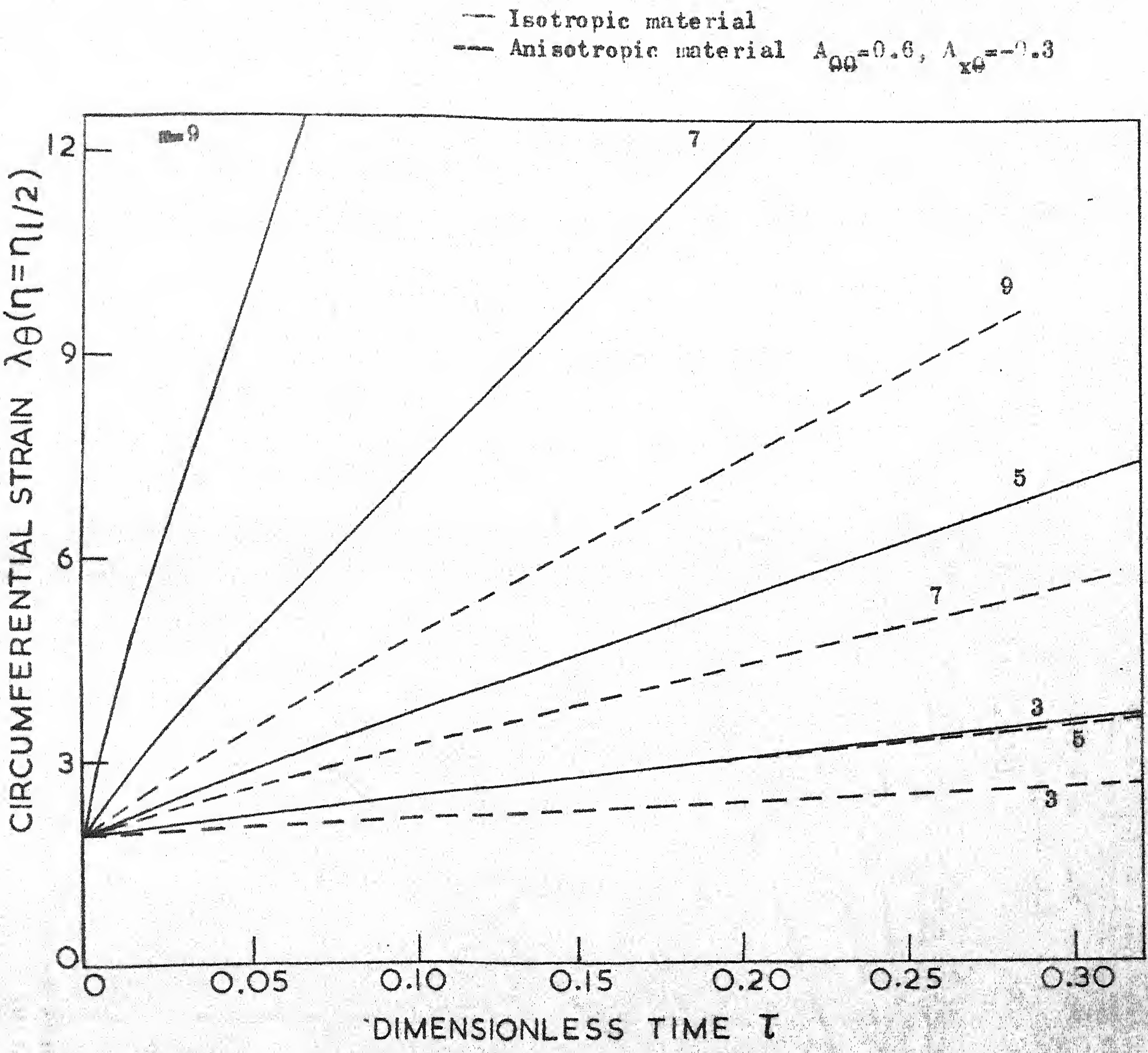


Figure 3.12 : Variation of circumferential strain at middle of simply supported cylindrical shell : influence of stress exponent m.

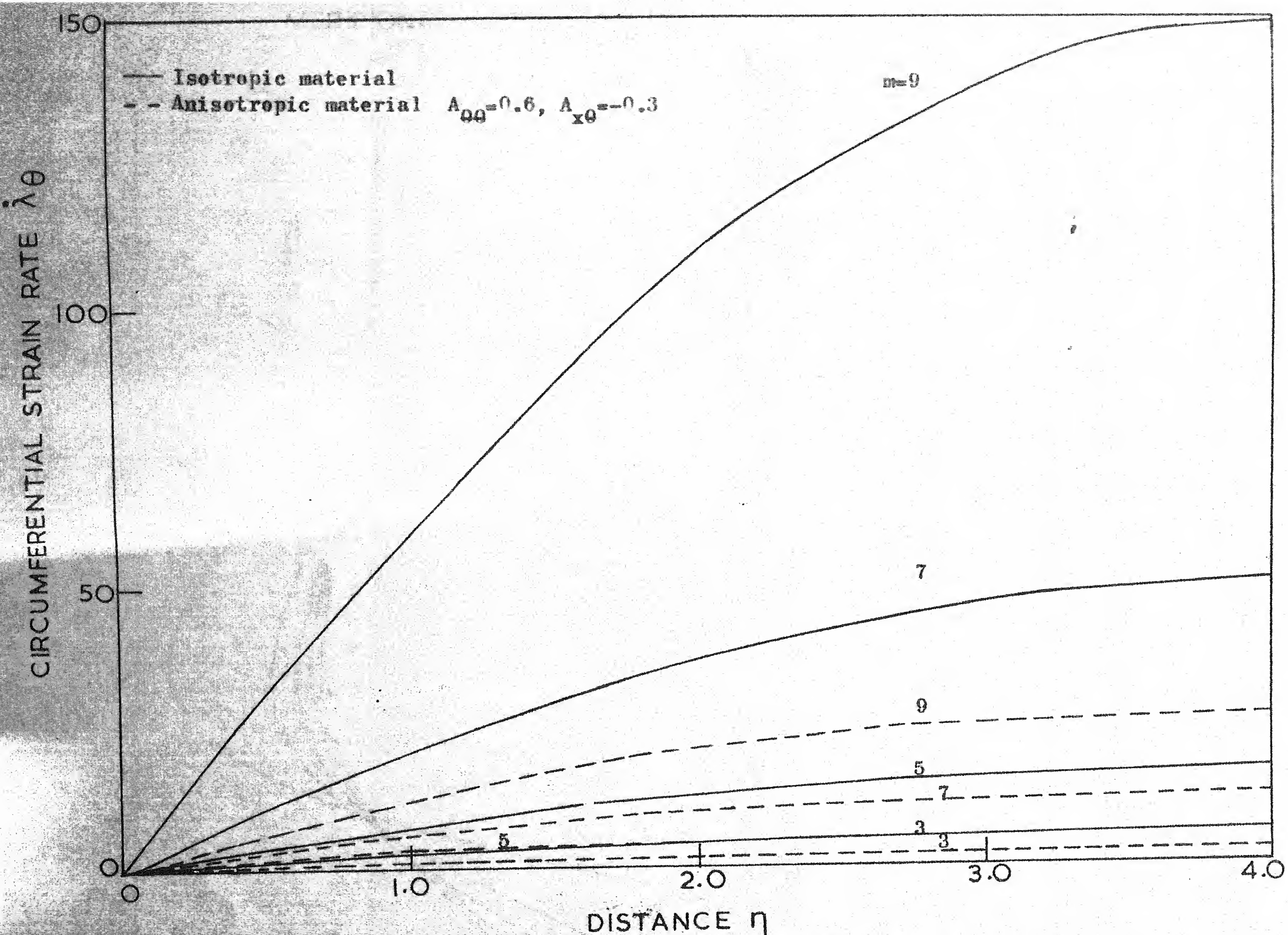


Figure 3.13 : Variation of circumferential strain rate in simply supported cylindrical shell at stationary state : influence of stress exponent m

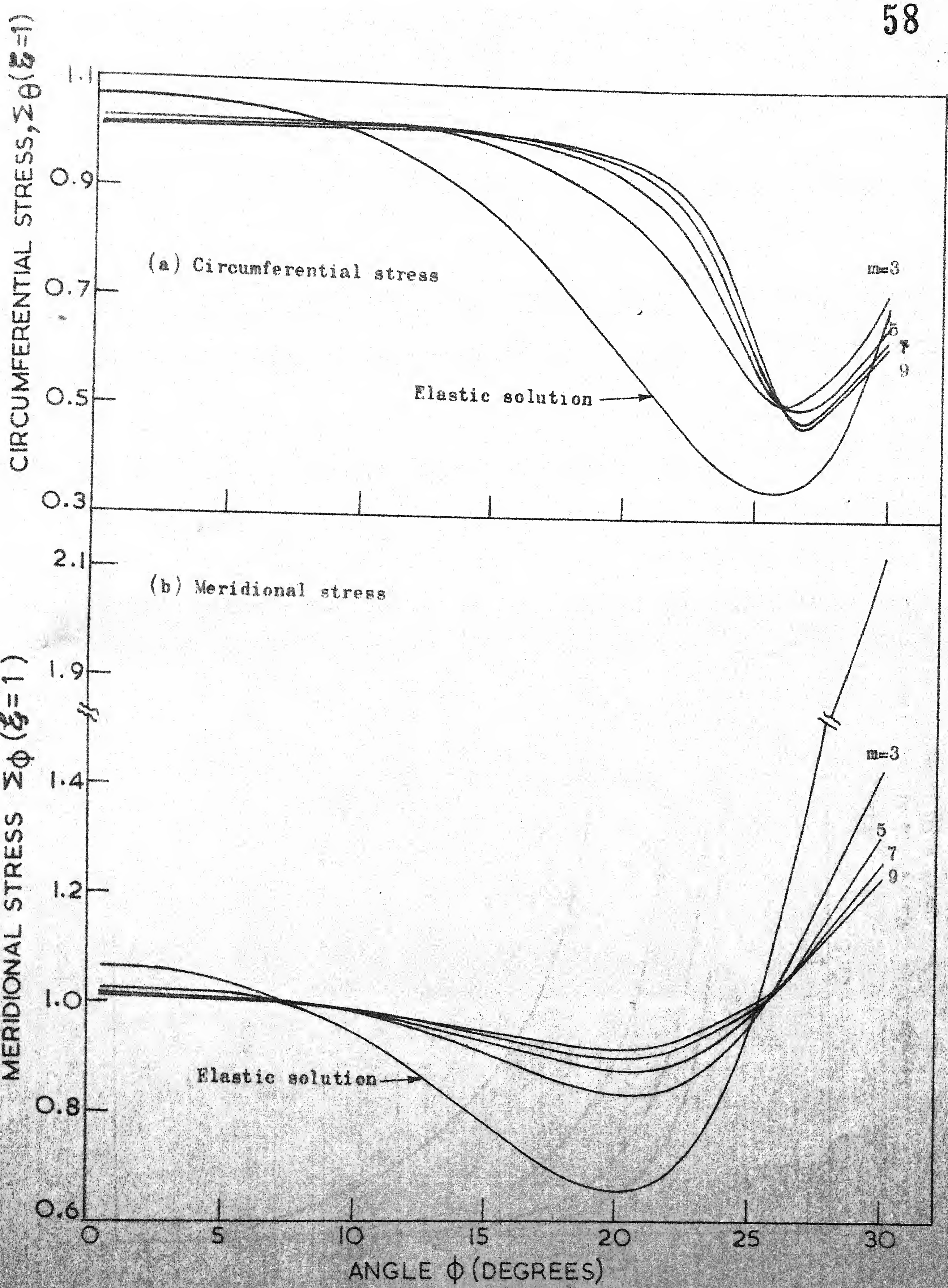


Figure 3.15 : Distribution of surface stress in clamped spherical shell at stationary state : influence of stress exponent m. Isotropic material

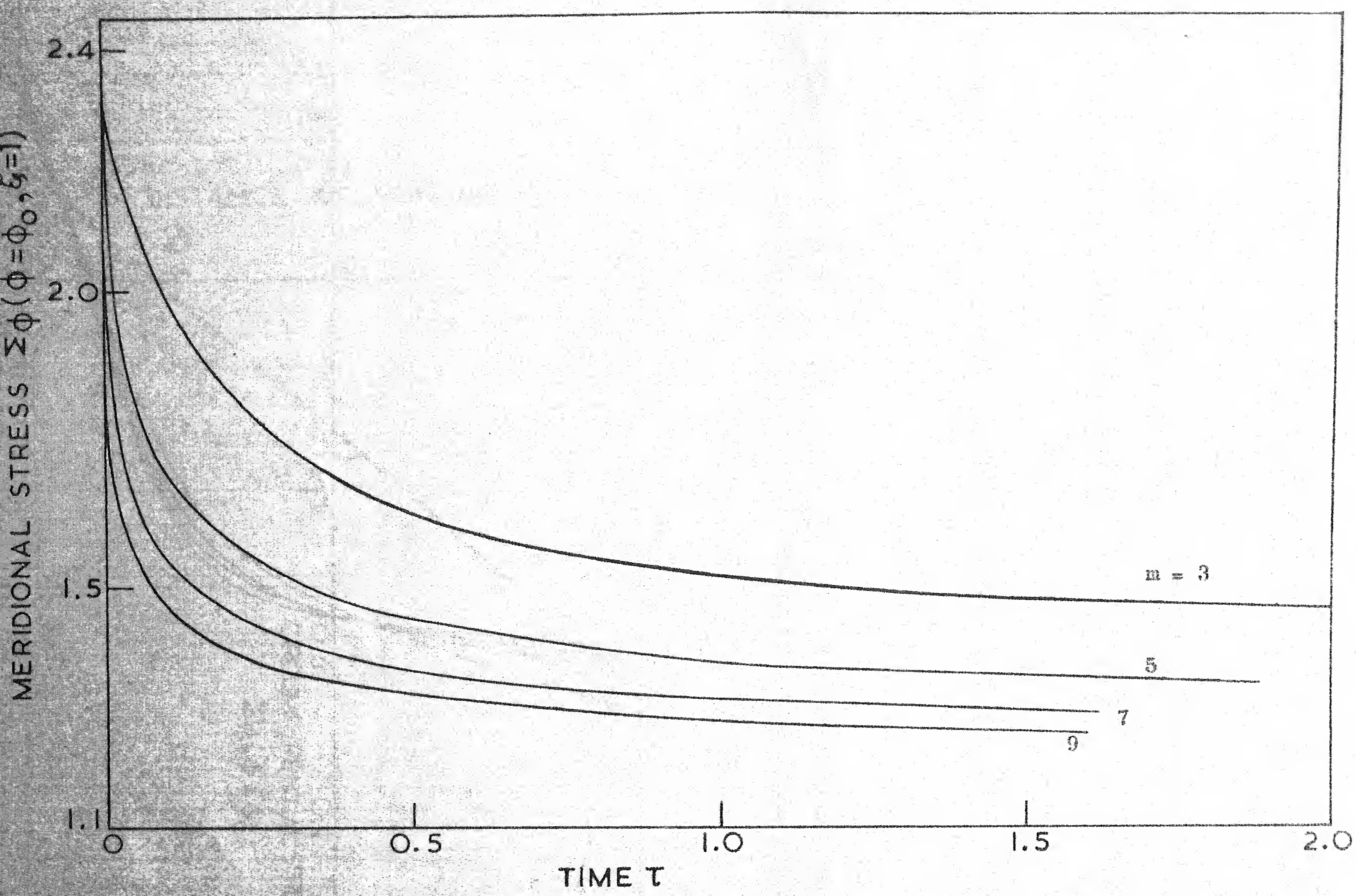


Figure 3.10 : Variation of meridional stress at clamped edge of spherical shell with time : influence of stress exponent m. Isotropic material.

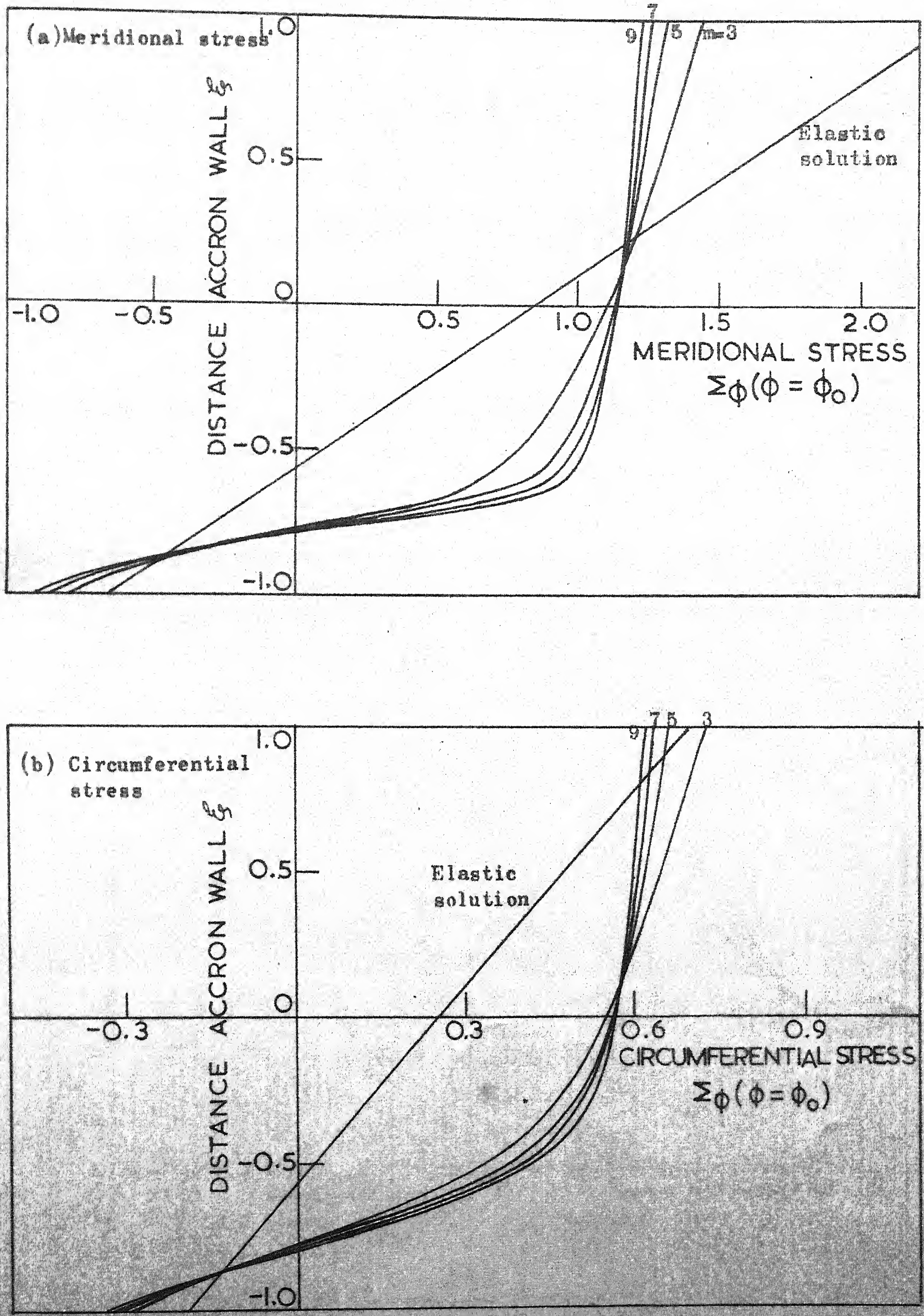


Figure 3.17 : Distribution of stress at clamped edge of spherical shell at stationary state : influence of stress exponent m. Isotropic material.

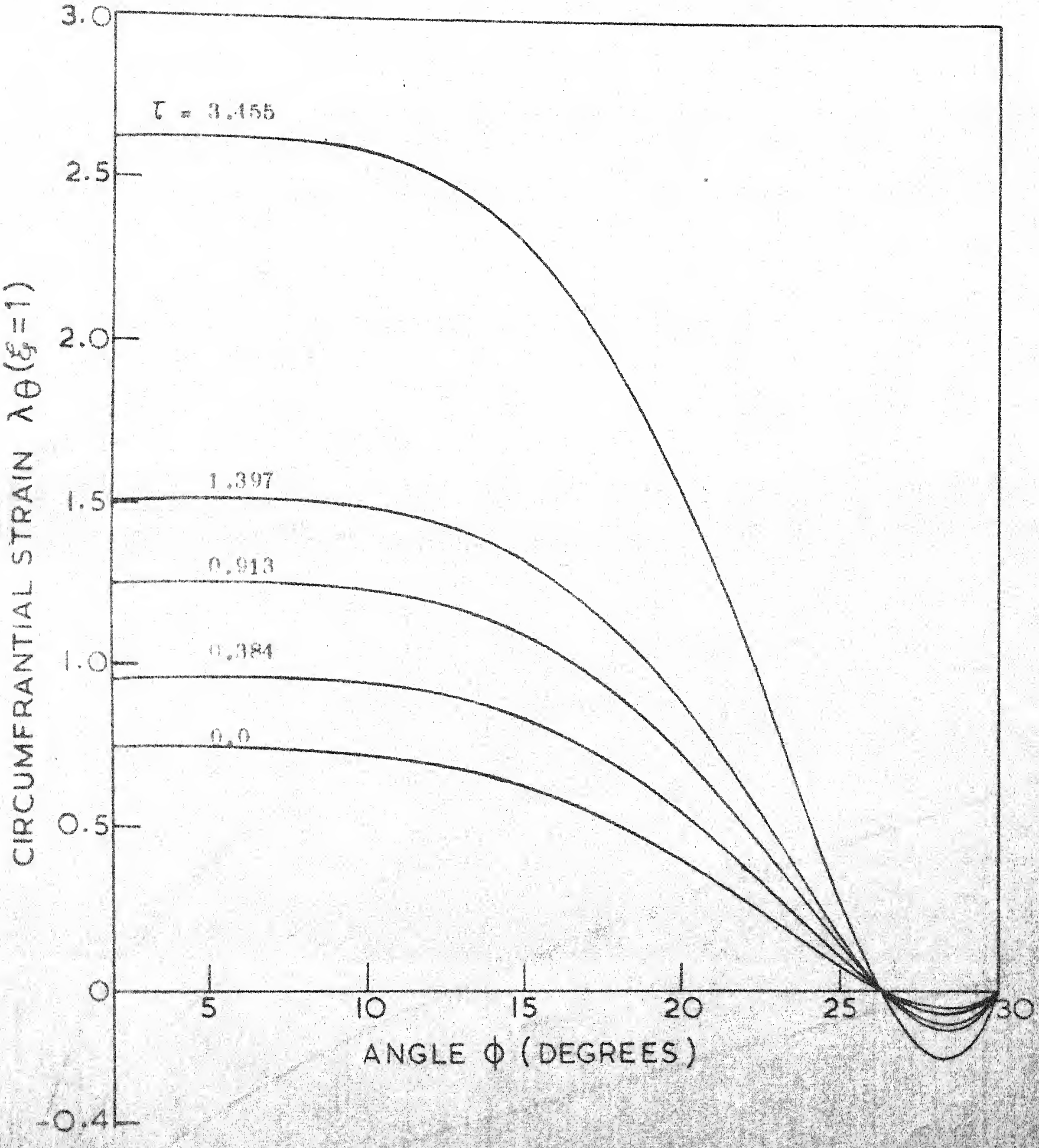


Figure 3.18 : Variation of circumferential strain in clamped spherical shell atstationary state :influence of stress exponent m . Isotropic material.

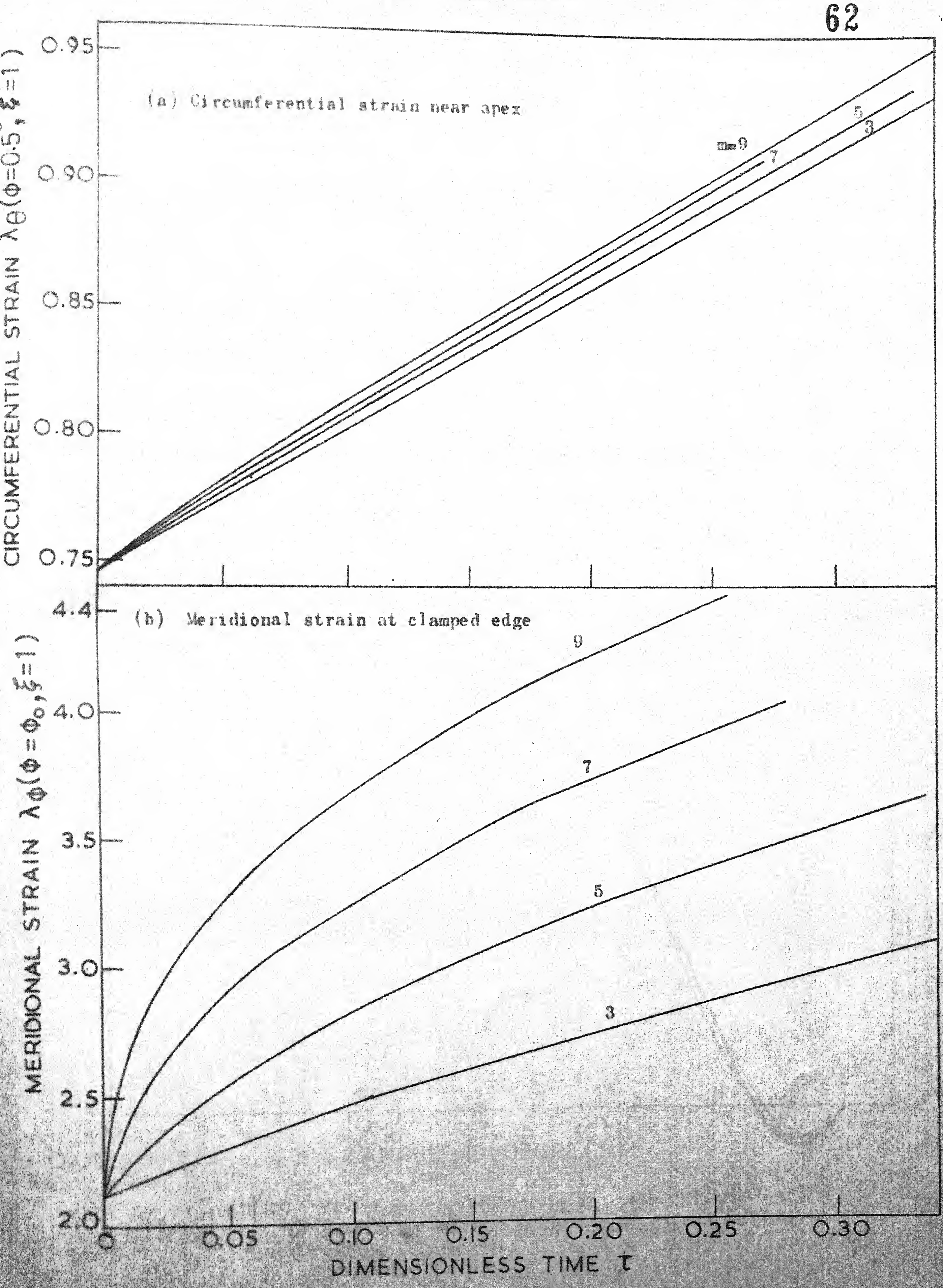


Figure 3.19 : Variation of strain in clamped spherical shell withtime influence of stress exponent m. Isotropic material.

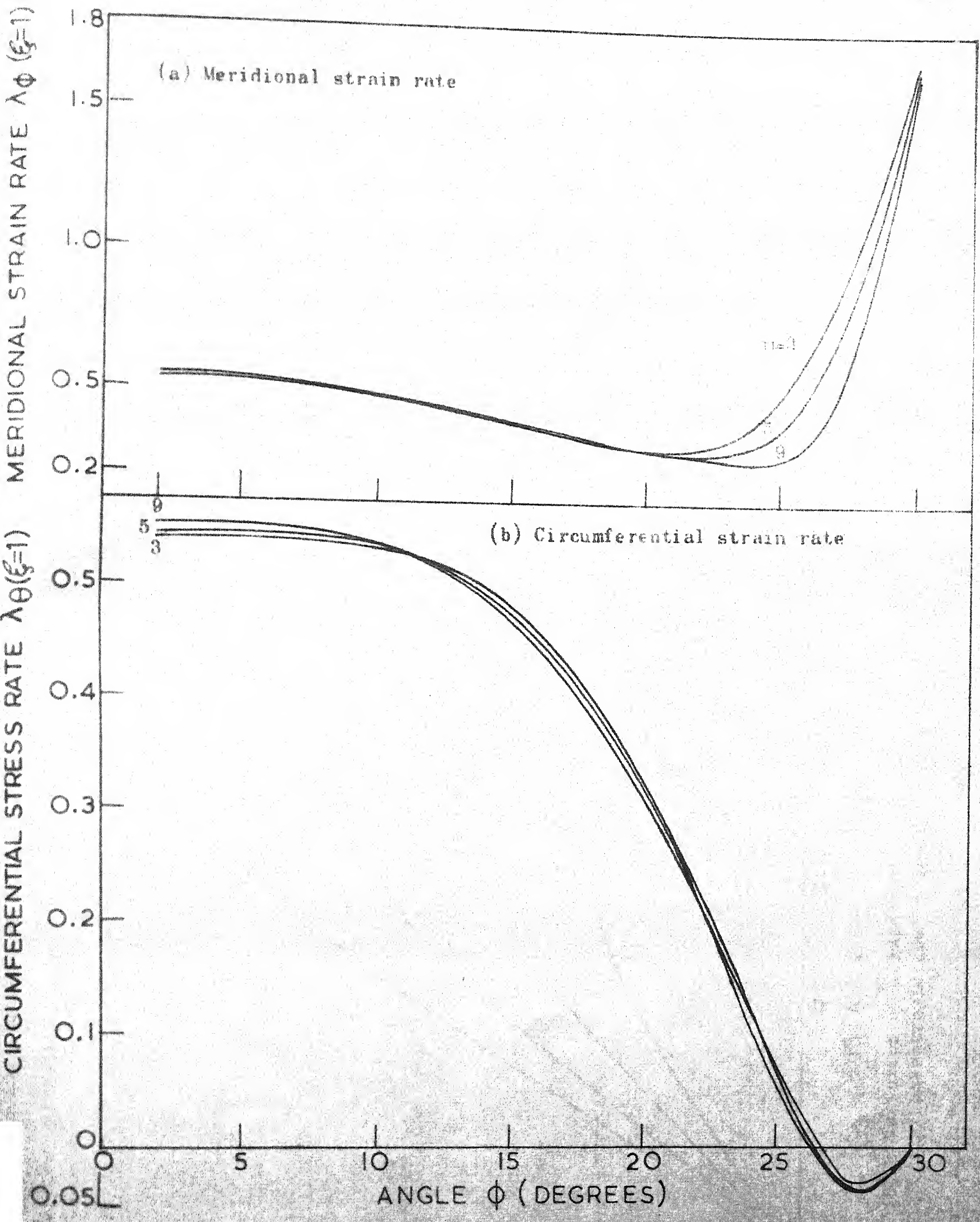


Figure 3.20 : Variation of strain rate in spherical shell at stationary state : influence of stress exponent m. Isotropic material.

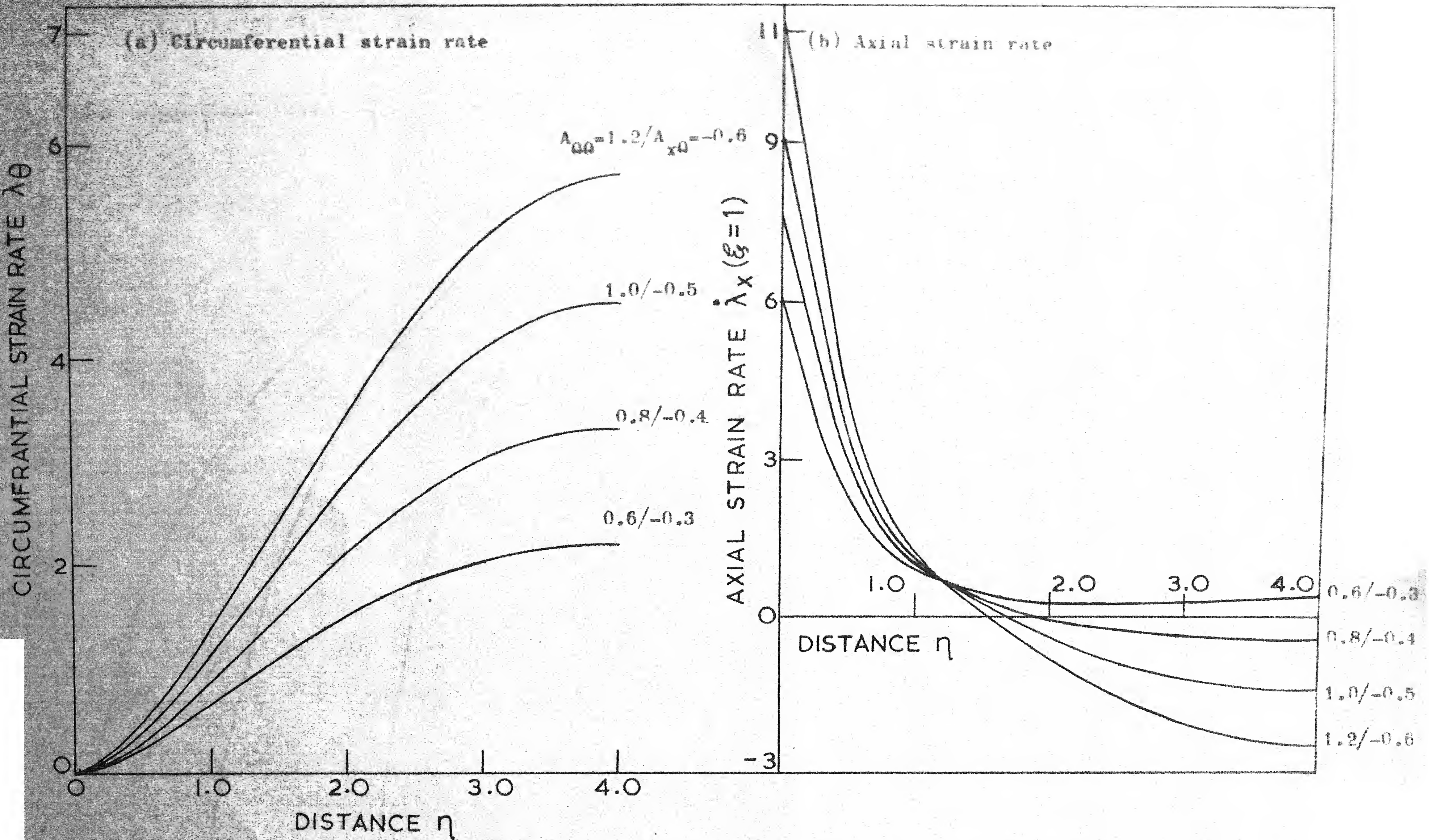


Figure 3.21 : Variation of strain rate in a clamped cylindrical shell at stationary state : influence of anisotropic coefficients. Stress exponent m = 3.

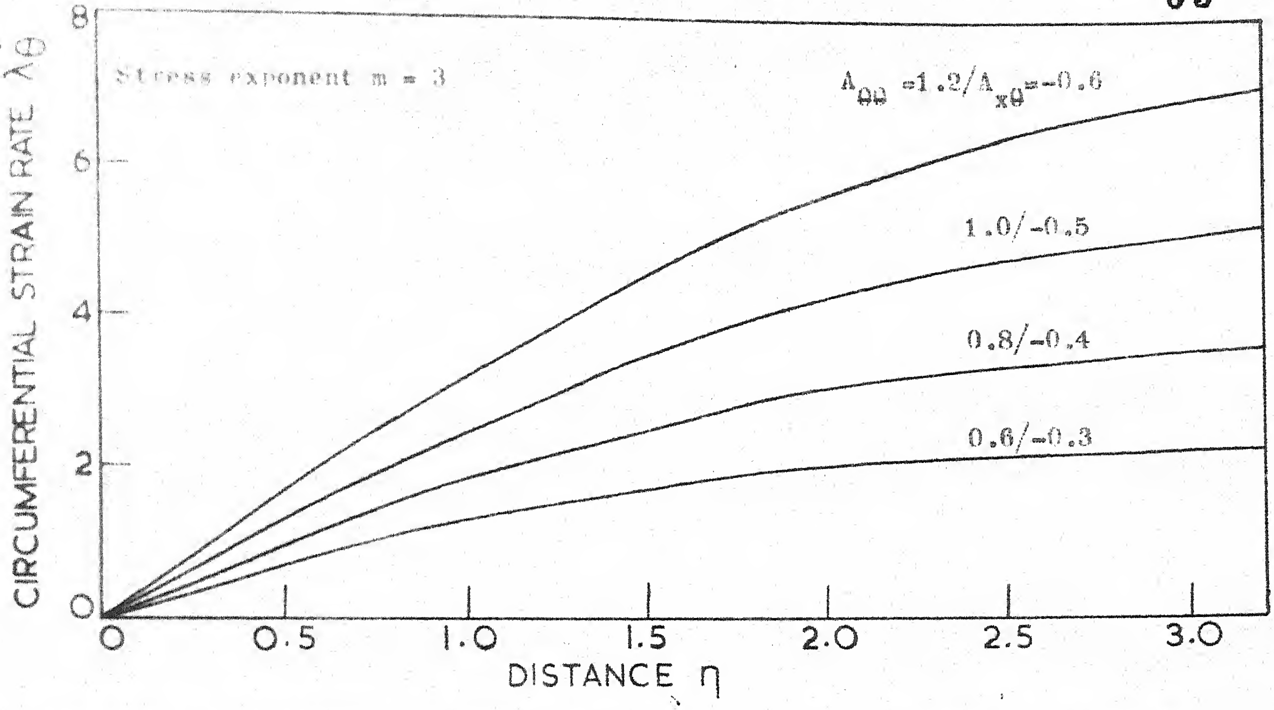


Figure 3.2 : Variation of circumferential strain rate in a simply supported cylindrical shell at stationary state : influence of anisotropy

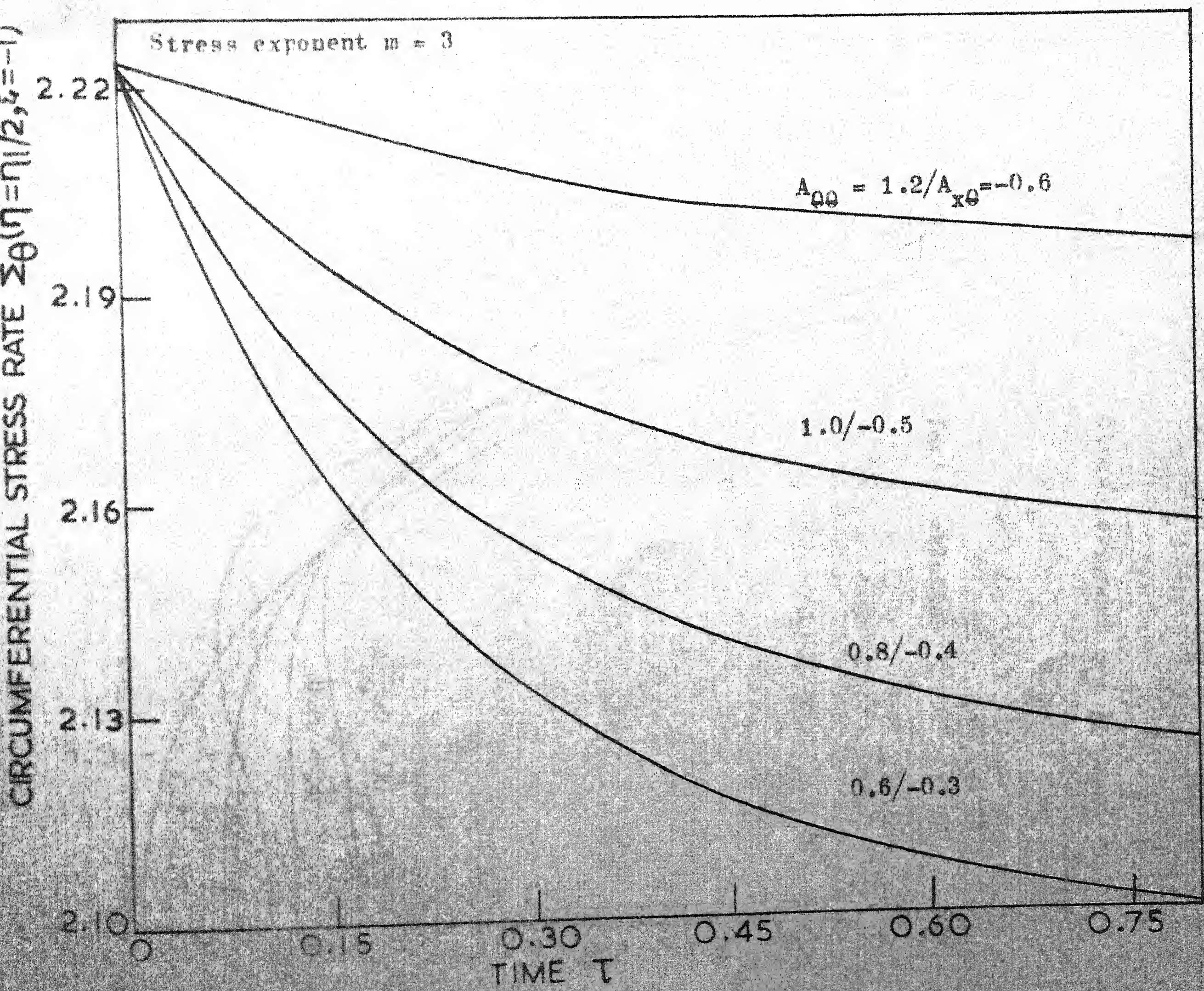


Figure 3.23 : Variation of circumferential stress in a simply supported cylindrical shell with time : effect ofanisotropy.

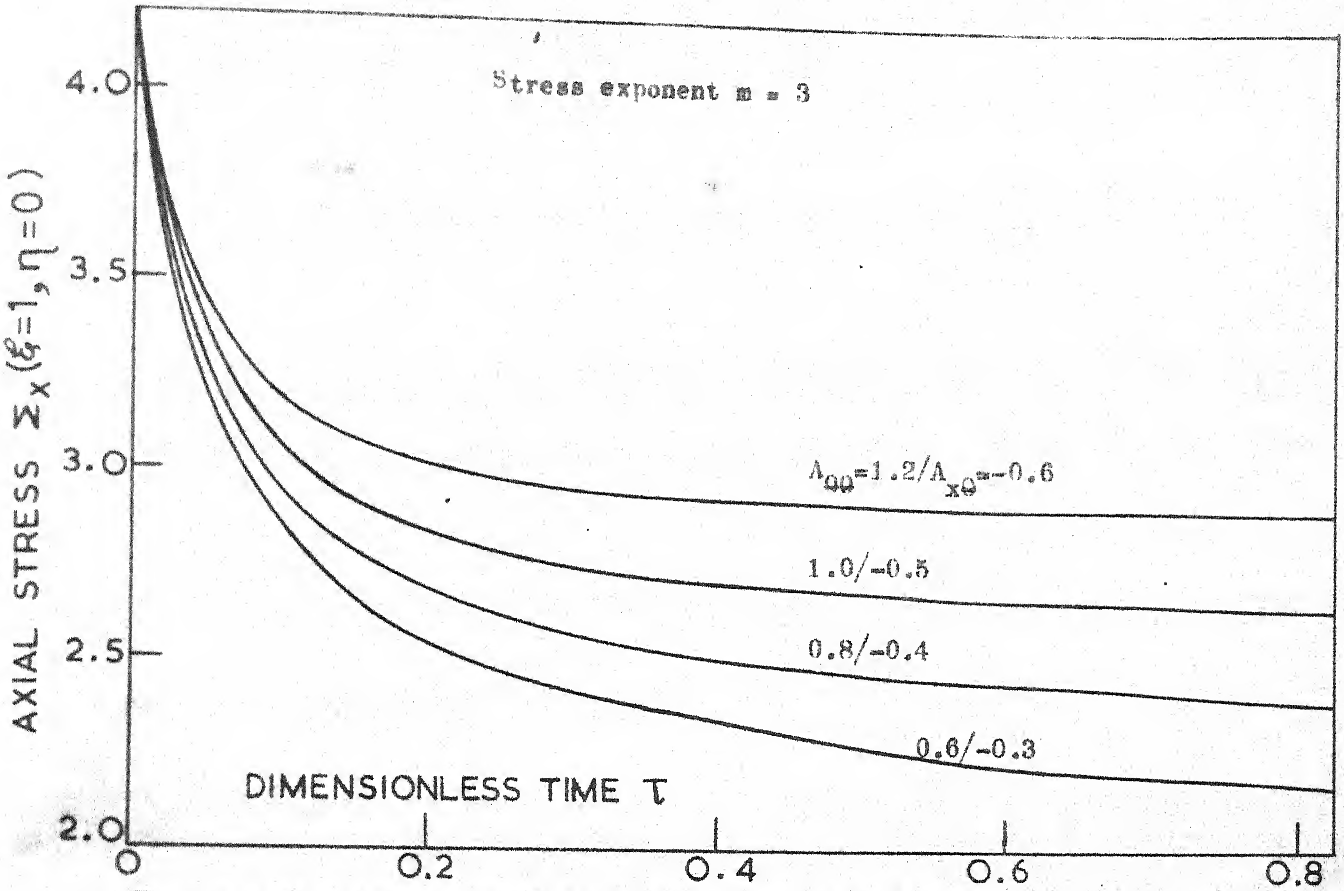


Figure 3.24 : Variation of axial stress at clamped edge of cylindrical shell with time : influence of anisotropy.

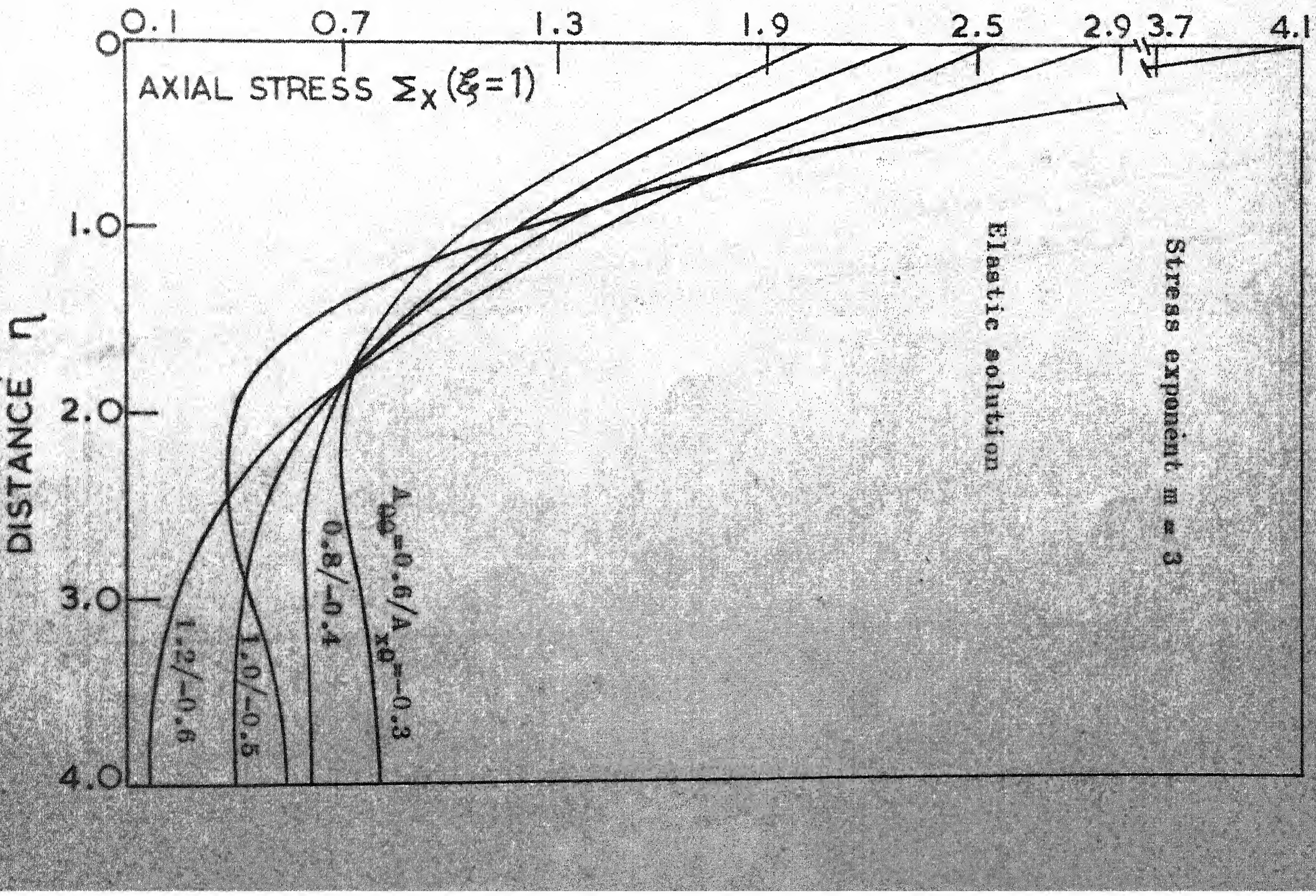
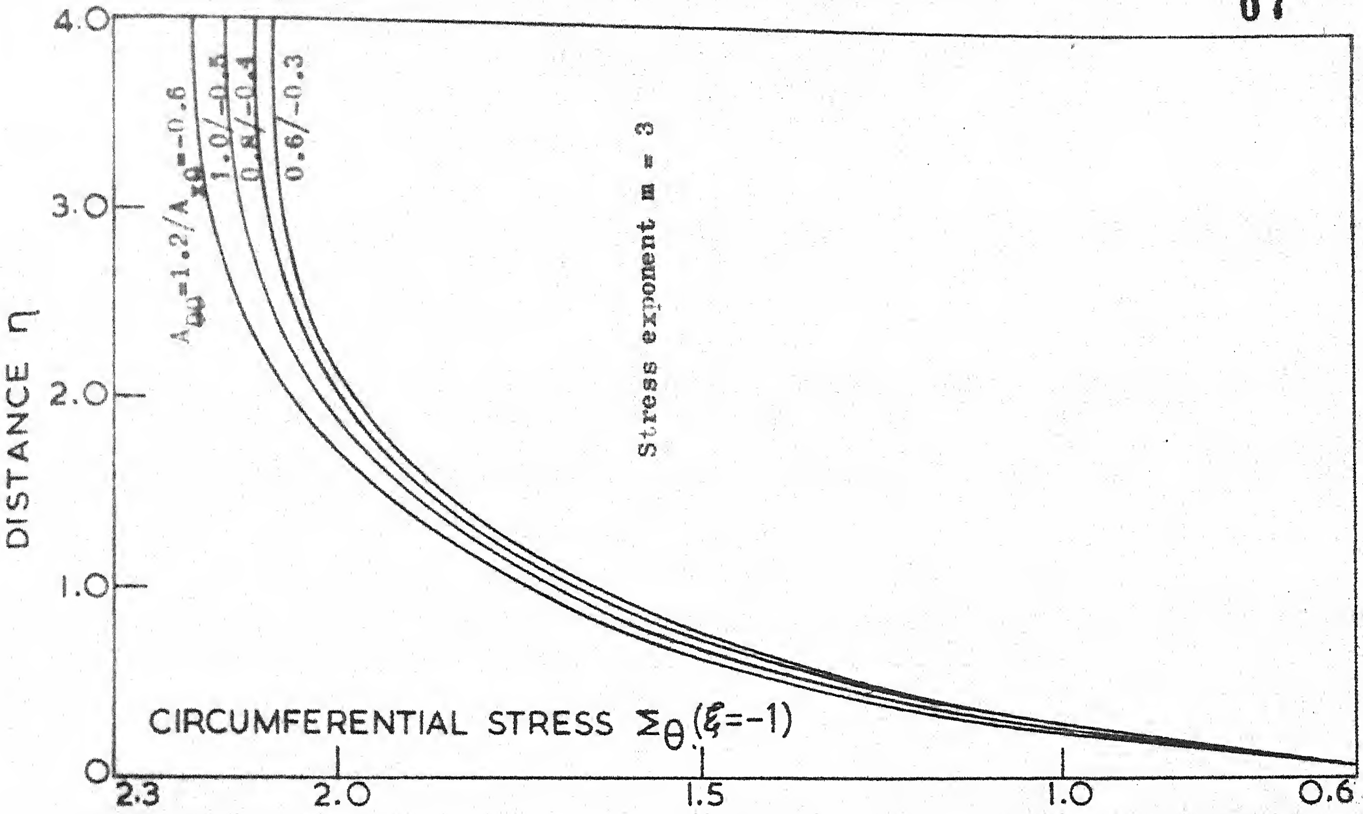


Figure 3.25 : Distribution of axial stress in clamped cylindrical shell at stationary state.



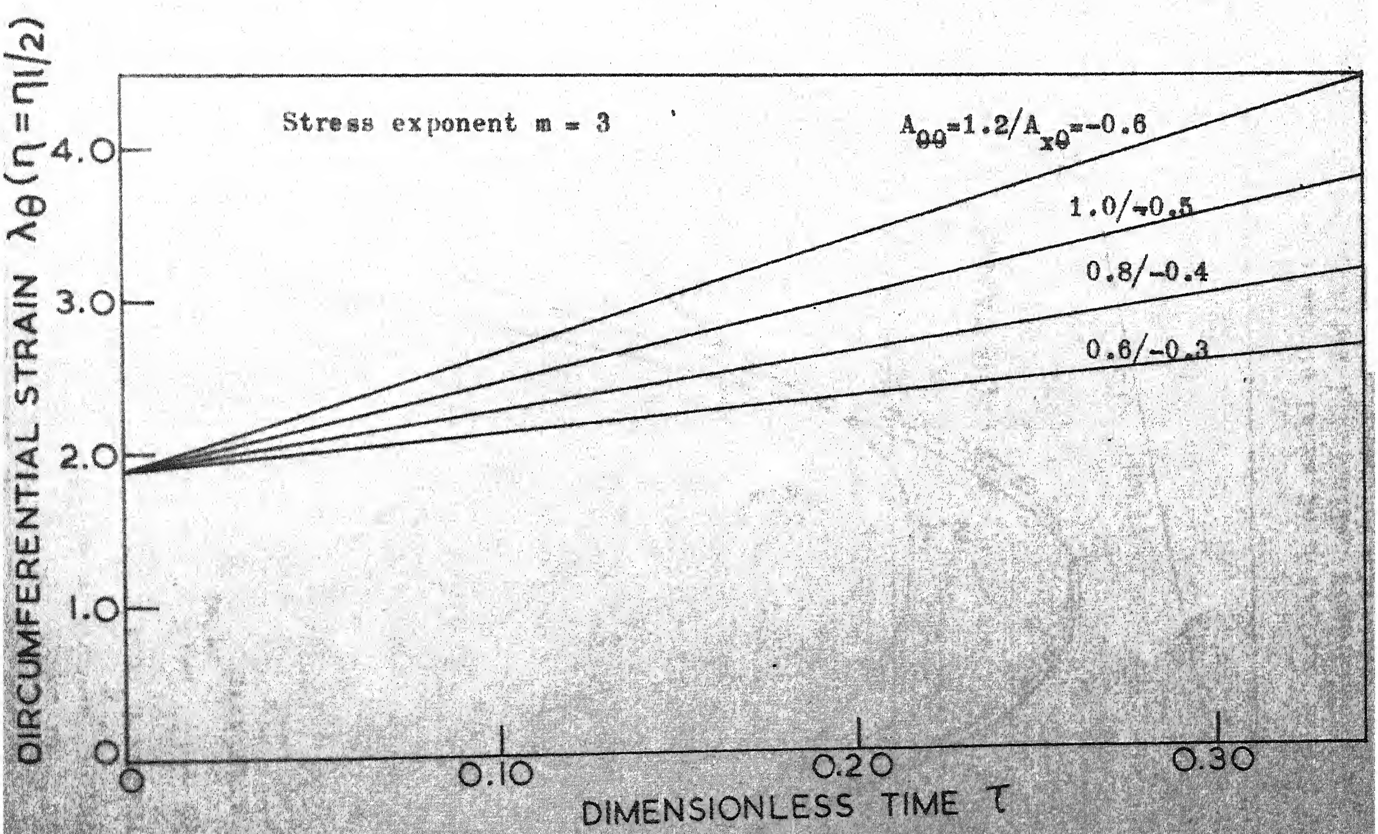


Figure 3.27 : Variation of circumferential strain at middle of simply supported cylindrical shell : influence of anisotropy.

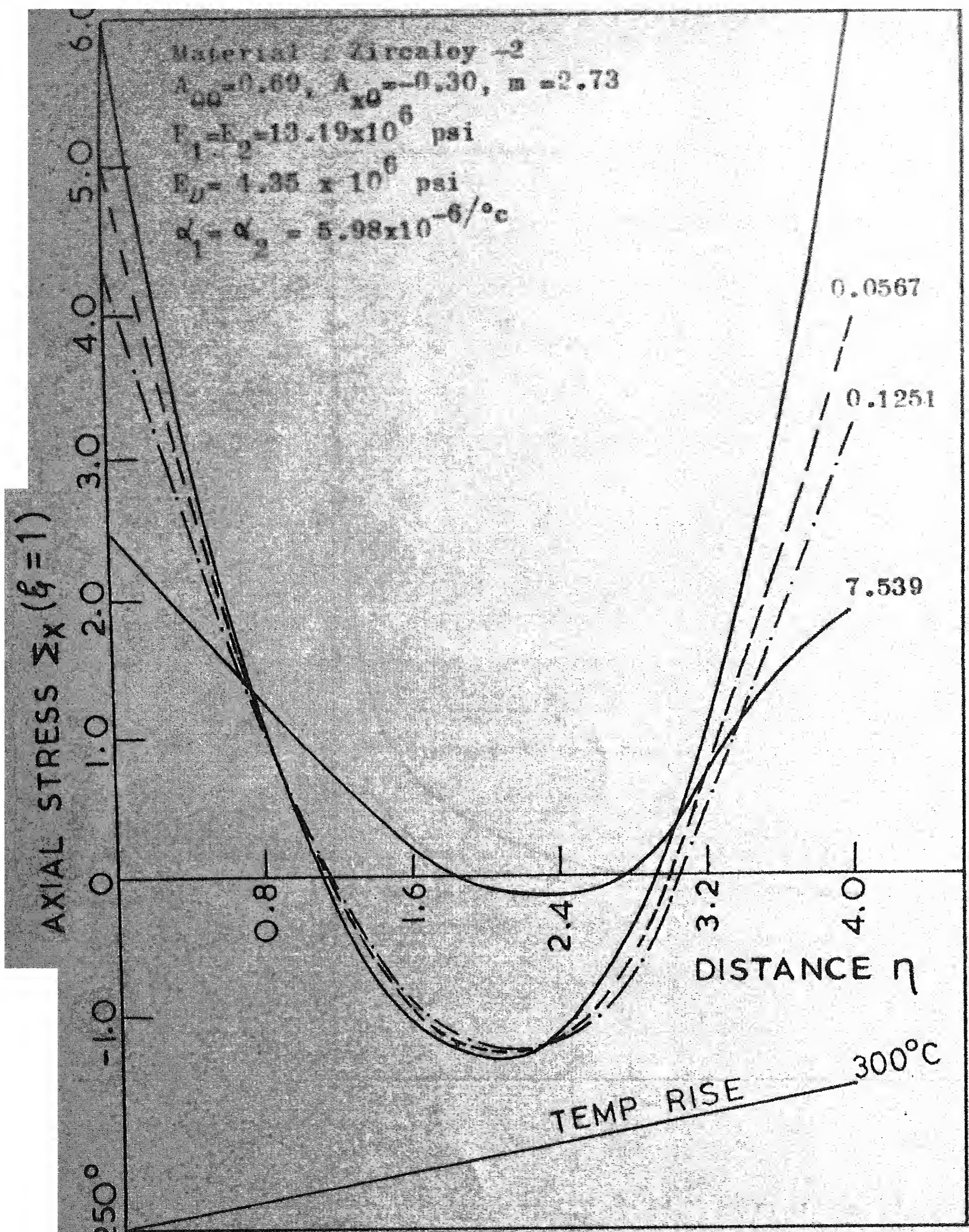


Figure 3.28 : Variation of axial stress in clamped cylindrical shell with linear temperature field

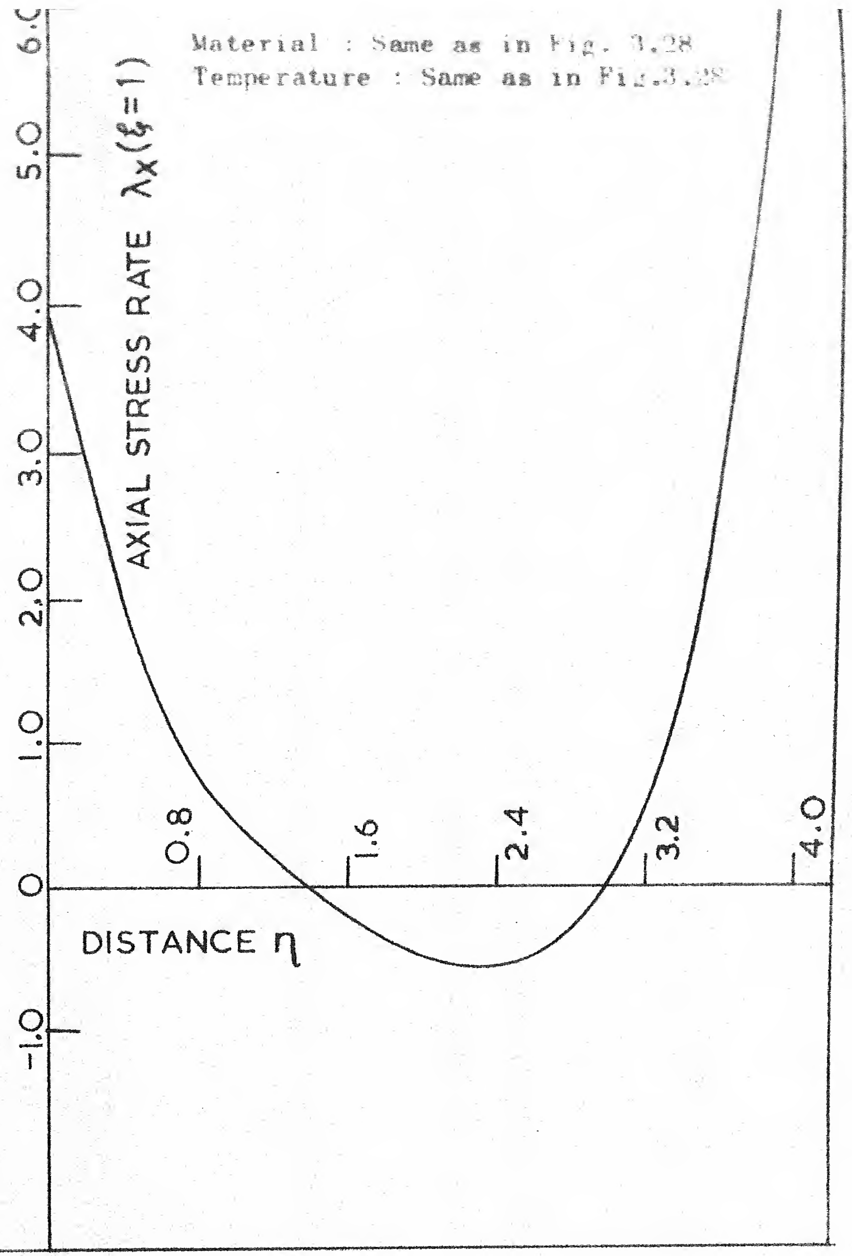


Fig.3.29 : Variation of axial strain rate in clamped cylindrical shell with linear temperature field

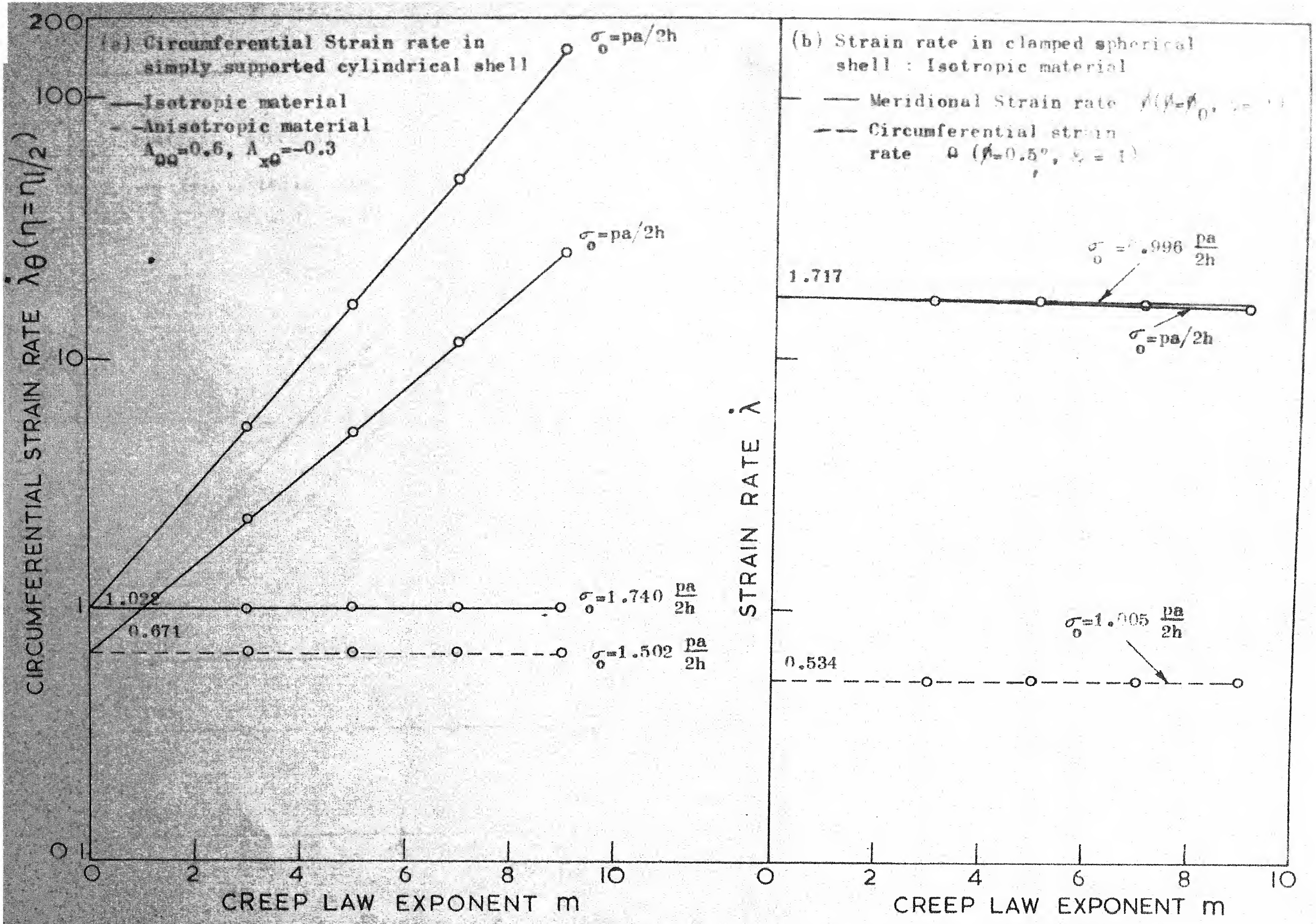


Figure 3.31 :Effect of different values of unit stress on rate of strain at stationary state.

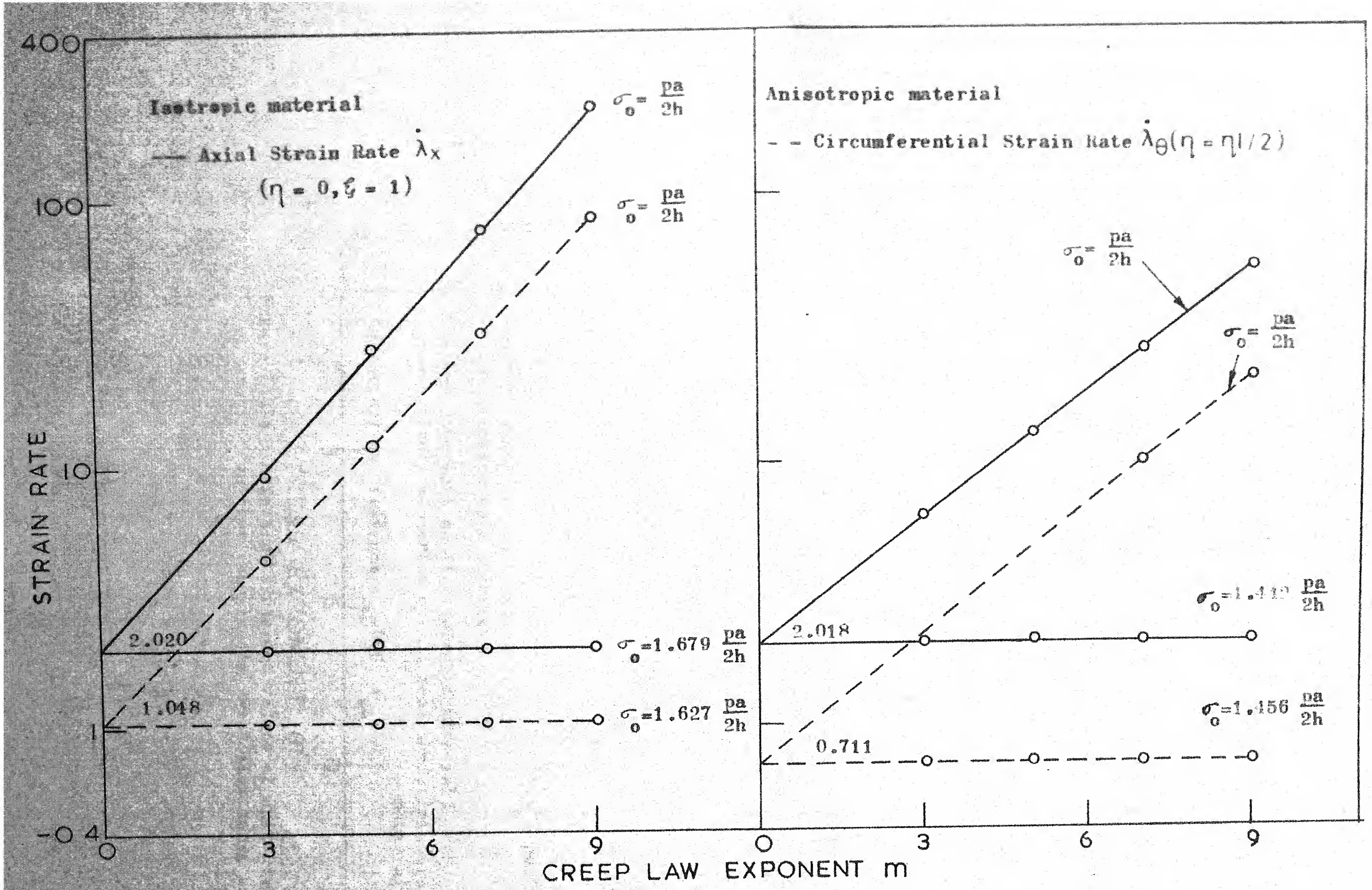


Figure 3.31(c) : Strain Rate in clamped Cylindrical Shell at stationary state

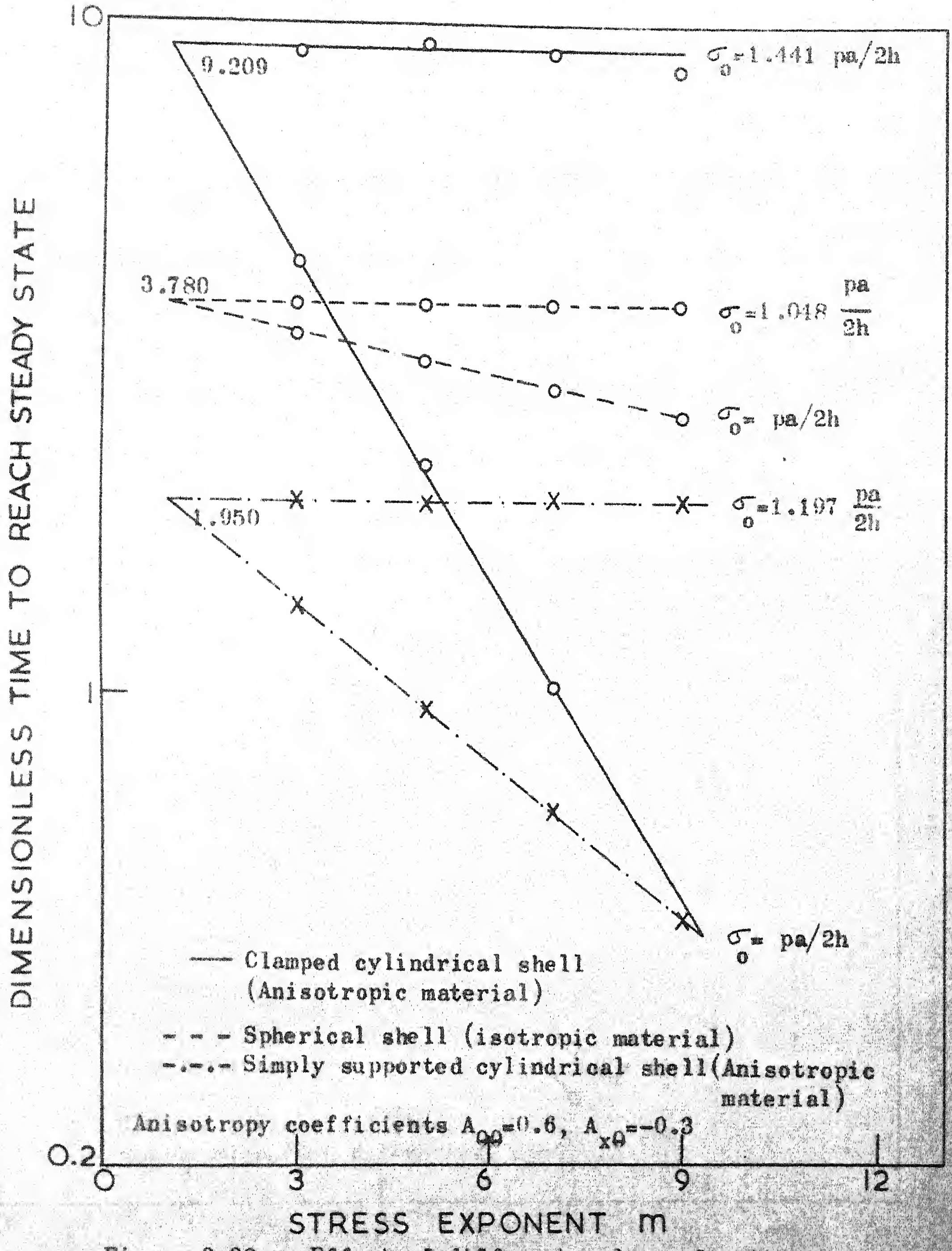


Figure 3.32a : Effect of different values of unit stress on time required to reach stationary state.

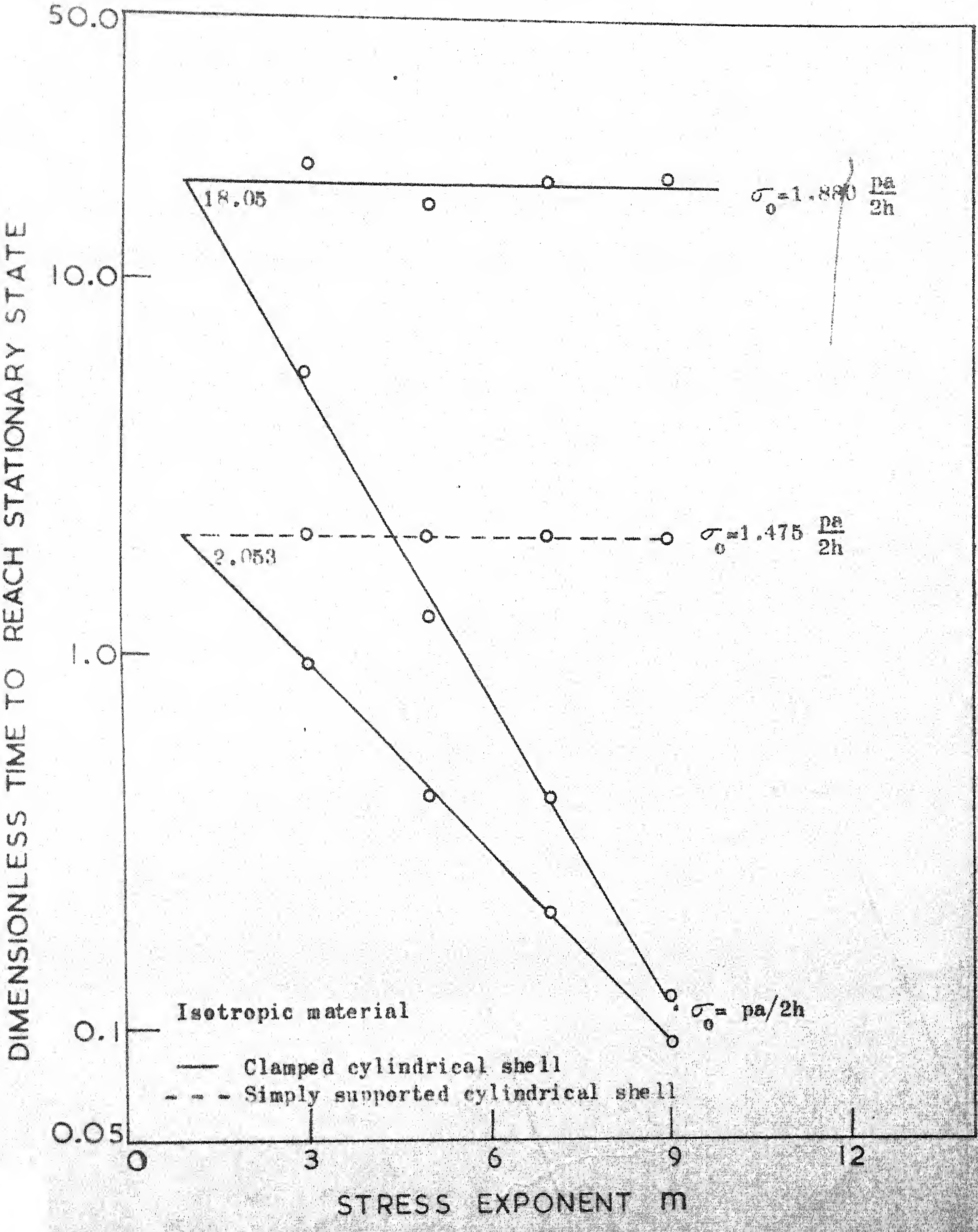


Figure 3.32b : Effect of different values of unit stress on time required to reach stationary state.

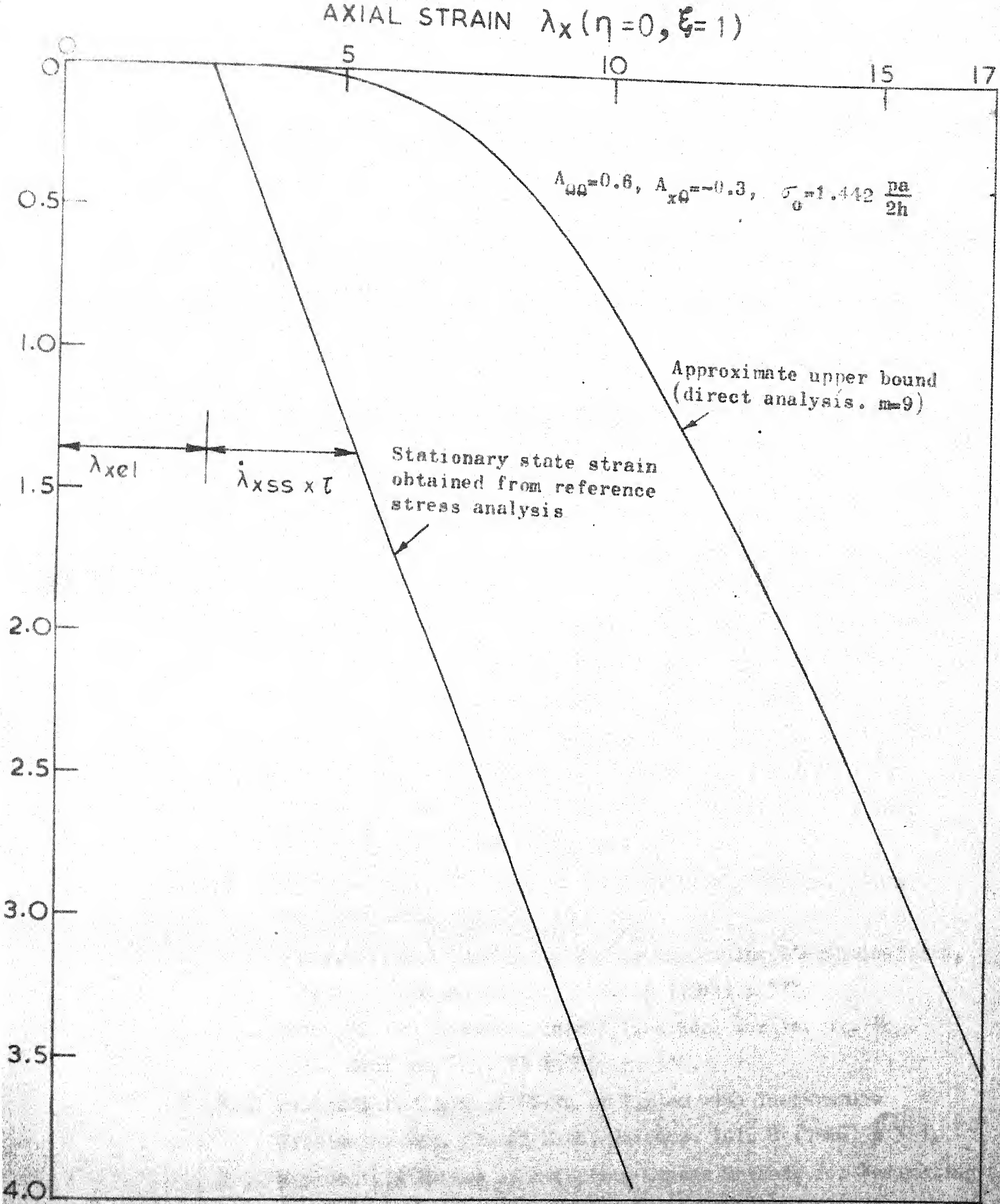


Figure 3.33 : Bounds on axial strain at clamped edge of cylindrical shell.

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APPENDIX A

MATRIX EQUATIONS FOR SHELLS

A.1 CLAMPED CIRCULAR CYLINDRICAL SHELL SUBJECTED TO UNIFORM PRESSURE

Shell length is divided into 2n equal parts. From the condition of symmetry about middle of length

$$\left.\begin{aligned} \gamma_n &= q_n = 0 \quad \text{at } \tau = 0 \\ \dot{\gamma}_n &= \dot{q}_n = 0 \quad \text{at } \tau \geqslant 0 \end{aligned}\right\} \qquad (A.1)$$

$$\left.\begin{aligned} q_{-1} &= q_1 - \frac{2\Delta}{\alpha}\left(2 - \frac{E_\nu}{E_1}\right) \\ \dot{q}_{-1} &= \dot{q}_1 - 2\Delta\ I_o \end{aligned}\right\} \qquad (A.2)$$

where

$$I_o = \frac{1}{2}\int_{-1}^{1} V_2\, d\xi \quad \text{at } \eta = 0 \qquad (A.3)$$

Subscripts 0, 1 .... n refer to node number 0, 1, 2....n.

$$\{X\} = \begin{Bmatrix} q_o \\ \gamma_{-1} \\ q_1 \\ \gamma_1 \\ \vdots \\ q_i \\ \gamma_i \\ \vdots \\ q_{n-1} \\ \gamma_{n-1} \\ q_{n+1} \\ \gamma_{n+1} \end{Bmatrix} \qquad (A.4)$$

$$B = \begin{Bmatrix} \frac{2\Delta}{\alpha}\left(2 - \frac{E_\nu}{E_1}\right) \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \qquad (A.5)$$

$$C = \begin{Bmatrix} \Delta^2\ \dot{F}_{c0} + 2\Delta\ I_o \\ \Delta^2\ \dot{G}_{c0} \\ \Delta^2\ \dot{F}_{c1} \\ \Delta^2\ \dot{G}_{c1} \\ \vdots \\ \Delta^2\ \dot{F}_{cn} \\ \Delta^2\ \dot{G}_{cn} \end{Bmatrix} \qquad (A.6)$$

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A.1 CLAMPED CIRCULAR CYLINDRICAL SHELL SUBJECTED TO UNIFORM PRESSURE

Shell length is divided into 2n equal parts. From the condition of symmetry about middle of length

$$\left.\begin{array}{lll} \gamma_n = q_n = 0 & & \text{at } \tau = 0 \\ \dot{\gamma}_n = \dot{q}_n = 0 & & \text{at } \tau \geqslant 0 \end{array}\right\} \tag{A.1}$$

$$\left.\begin{array}{l} q_{-1} = q_1 - \dfrac{2\Delta}{\alpha}\left(2 - \dfrac{E_\nu}{E_1}\right) \\ \dot{q}_{-1} = \dot{q}_1 - 2\Delta\ I_o \end{array}\right\} \tag{A.2}$$

where

$$I_o = \frac{1}{2}\int_{-1}^{1} V_2 \, d\xi \quad \text{at} \quad \eta = 0 \tag{A.3}$$

Subscripts 0, 1 .... n refer to node number 0, 1, 2....n.

$$\{X\} = \begin{Bmatrix} q_o \\ \gamma_{-1} \\ q_1 \\ \gamma_1 \\ \vdots \\ q_i \\ \gamma_i \\ \vdots \\ q_{n-1} \\ \gamma_{n-1} \\ q_{n+1} \\ \gamma_{n+1} \end{Bmatrix} \tag{A.4}$$

$$B = \begin{Bmatrix} \dfrac{2\Delta}{\alpha}\left(2 - \dfrac{E_\nu}{E_1}\right) \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \tag{A.5}$$

$$C = \begin{Bmatrix} \Delta^2\ \dot{F}_{c0} + 2\Delta\ I_o \\ \Delta^2\ \dot{G}_{c0} \\ \Delta^2\ \dot{F}_{c1} \\ \Delta^2\ \dot{G}_{c1} \\ \vdots \\ \Delta^2\ \dot{F}_{cn} \\ \Delta^2\ \dot{G}_{cn} \end{Bmatrix} \tag{A.6}$$

$$[A] = \begin{bmatrix} -2 & 0 & 2 \\ s & 1 & 0 & 1 \\ 1 & 0 & -2 & b & 1 \\ & 0 & s & -2 & 0 & 1 \\ & & 1 & 0 & -2 & b & 1 \\ & & & 1 & s & -2 & 0 & 1 \\ & & & & & \ddots \\ & & & & & 1 & 0 & -2 & b & 1 \\ & & & & & & 1 & s & -2 & 0 & 1 \\ & & & & & & & & \ddots \\ & & & & & & & & 1 & 0 & -2 & b & 0 \\ & & & & & & & & & 1 & s & -2 & 0 & 0 \\ & & & & & & & & & & 1 & 0 & 1 & 0 \\ & & & & & & & & & & & 1 & 0 & 1 \end{bmatrix} \quad (A.7)$$

where $s = \Delta^2$ and $b = -\Delta^2$

A.2 SIMPLY SUPPORTED CYLINDRICAL SHELL SUBJECTED TO UNIFORM PRESSURE

$$\{X\} = \begin{Bmatrix} q_o \\ \gamma_o \\ q_1 \\ \gamma_1 \\ \vdots \\ \vdots \\ q_{n-1} \\ \gamma_{n-1} \\ q_{n+1} \\ \gamma_{n+1} \end{Bmatrix} \quad (A.8)$$

$$B = \begin{Bmatrix} \frac{2\Delta}{\alpha}\left(2 - \frac{E_\nu}{E_1}\right) \\ 0 \\ \vdots \end{Bmatrix} \quad (A.9)$$

$$C = \begin{Bmatrix} \Delta^2 \dot{F}_{c0} + 2\Delta I_o \\ \Delta^2 \dot{G}_{c0} - 2\Delta I_1 \\ \Delta^2 \dot{F}_{c1} \\ \Delta^2 \dot{G}_{c1} \\ \vdots \\ \Delta^2 F_{cn} \\ \Delta^2 G_{cn} \end{Bmatrix} \quad (A.10)$$

$$I_1 = \left(\frac{3E^*}{E_1}\right)^{1/2} \tfrac{1}{2} \left(\int_{-1}^{1} U_1 \xi_1 \, d\xi\right) \left.\frac{A_{xx} E_1 + E_\nu A_{x\theta}}{E^*}\right|_{\eta=0} \quad (A.11)$$

$$[A]=\begin{bmatrix}
-2 & s & 2 & & & & & & & & & & \\
s & -2 & 0 & 2 & & & & & & & & & \\
1 & 0 & -2 & b & 1 & & & & & & & & \\
 & 1 & s & -2 & 0 & 1 & & & & & & & \\
 & & & \ddots & & & & & & & & & \\
 & & & 1 & 0 & -2 & b & 1 & & & & & \\
 & & & & 1 & s & -2 & 0 & 1 & & & & \\
 & & & & & & & \ddots & & & & & \\
 & & & & & & & 1 & 0 & -2 & b & 0 & \\
 & & & & & & & & 1 & s & -2 & 0 & 0 \\
 & & & & & & & & & 1 & 0 & 1 & 0 \\
 & & & & & & & & & & 1 & 0 & 1
\end{bmatrix} \qquad (A.12)$$

$$\left.\begin{aligned}
q_{-1} &= q_1 - \frac{2\Delta}{\alpha}\left(2 - \frac{E_\nu}{E_1}\right) \\
\dot{q}_{-1} &= \dot{q}_1 - 2\Delta I_o \\
\gamma_{-1} &= \gamma_1 \\
\dot{\gamma}_{-1} &= \dot{\gamma}_1 + 2\Delta I_1
\end{aligned}\right\} \qquad (A.13)$$

A.3 CLAMPED SPHERICAL SHELL SUBJECTED TO UNIFORM PRESSURE

Shell meridian is divided into n parts from $\phi = 0$ to $\phi = \phi_0$.

$$\left.\begin{aligned}
a_i &= 1 - \cot \phi_i \frac{\Delta}{2} \\
b_i &= -2 - \frac{E_2}{E_1} \Delta^2 \cot^2 \phi_i + \frac{E_2}{E_1} \Delta^2 \\
c_i &= 1 + \cot \phi_i \frac{\Delta}{2} \\
d_i &= 2k^2 \Delta^2 \\
e_i &= -2 - \frac{E_2}{E_1} \Delta^2 \cot^2 \phi_i - \frac{E_2}{E_1} \Delta^2
\end{aligned}\right\} \qquad (A.14)$$

$$[A]=\begin{bmatrix}
b_1 & -d_1 & c_1 &&&&&&&&&& \\
d_1 & e_1 & 0 & c_1 &&&&&&&&& \\
a_2 & 0 & b_2 & -d_2 & c_2 &&&&&&&& \\
& a_2 & d_2 & e_2 & 0 & c_2 &&&&&&& \\
&&&&& \ddots &&&&&&& \\
&&&& a_i & 0 & b_i & -d_i & c_i &&&& \\
&&&&& a_i & d_i & e_i & 0 & c_i &&& \\
&&&&&&&& \ddots &&&& \\
&&&&&&& a_{n-1} & 0 & b_{n-1} & -d_{n-1} & c_{n-1} & \\
&&&&&&&& a_{n-1} & d_{n-1} & e_{n-1} & 0 & 0 \\
&&&&&&&&& A_n & 0 & B_n & 0 \\
&&&&&&&&&& a_n & d_n & c_n
\end{bmatrix} \qquad (A.15)$$

where $A_n = a_n + c_n$

$$B_n = b_n + 2\,\frac{E_\nu}{E_1}\cot\phi_n\,\Delta\,c_n \qquad (A.16)$$

$$\{X\}=\begin{Bmatrix} S_1 \\ \theta_1 \\ S_2 \\ \theta_2 \\ \vdots \\ \vdots \\ S_{n-1} \\ \theta_{n-1} \\ S_n \\ \theta_{n+1} \end{Bmatrix} \qquad (A.17)$$

$$\{B\}=\begin{Bmatrix} 0 \\ 0 \\ \vdots \\ -4\,c_n\,\dfrac{\Delta k^2}{\alpha}\left(1-\dfrac{E_\nu}{E_1}\right) \\ 0 \end{Bmatrix} \qquad (A.18)$$

$$\{C\}=\begin{Bmatrix} 2k^2\,\Delta^2\,\dot{F}_{c1} \\ 2k^2\,\Delta^2\,\dot{G}_{c1} \\ \vdots \\ 2k^2\,\Delta^2\,\dot{F}_{cn} - 4\Delta k^2\,I_n\,c_n \\ 2k^2\,\Delta^2\,\dot{G}_{cn} \end{Bmatrix} \qquad (A.19)$$

$$I_n = \tfrac{1}{2}\int_{-1}^{1} V_2\,d\xi\,\Big|_{\eta=\eta_n} \qquad (A.20)$$

$$
\left.
\begin{aligned}
S_{n+1} &= S_{n-1} + 4\,\frac{\Delta k^2}{\alpha}\left(1-\frac{E_\nu}{E_1}\right) + 2\Delta\frac{E_\nu}{E_1}\cot\phi_n\,S_n \\
\dot{S}_{n+1} &= \dot{S}_{n-1} + 4\,\Delta\,k^2\,I_n + 2\,\Delta\,\frac{E_\nu}{E_1}\cot\phi_n\,\dot{S}_n
\end{aligned}
\right\}
\qquad (A.21)
$$

A.4 CLAMPED CIRCULAR CYLINDRICAL SHELL SUBJECTED TO UNIFORM PRESSURE AND NON UNIFORM TEMPERATURE

Shell axis is divided into n equal parts.

$$
[A] = \begin{bmatrix}
-2 & 0 & 2 & & & & & & & & & & & & \\
s & 1 & 0 & 1 & & & & & & & & & & & \\
1 & 0 & -2 & b & 1 & & & & & & & & & & \\
 & 0 & s & -2 & 0 & 1 & & & & & & & & & \\
 & & 1 & 0 & -2 & b & 1 & & & & & & & & \\
 & & & 1 & s & -2 & 0 & 1 & & & & & & & \\
 & & & & & & & \ddots & & & & & & & \\
 & & & & & & & & & 1 & 0 & -2 & b & 1 & \\
 & & & & & & & & & & 1 & s & -2 & 0 & 0 \\
 & & & & & & & & & & & 1 & 0 & -1 & 0 \\
 & & & & & & & & & & & & 1 & s & 1
\end{bmatrix}
\qquad (A.22)
$$

$$\{B\} = \begin{Bmatrix} 2\Delta\left(2 - \frac{E_\nu}{E_1}\right) + (T'_1 + T'_o)\Delta \\ 0 \\ \frac{T'_2 - T'_0}{2}\Delta \\ 0 \\ \vdots \\ \frac{T'_{i+1} - T'_{i-1}}{2}\Delta \\ 0 \\ \vdots \\ \frac{T'_n - T'_{n-2}}{2}\Delta \\ 0 \\ -\Delta\left(2 - \frac{E_\nu}{E_1}\right) - \frac{\Delta}{2}(T'_n + T'_{n-1}) \\ 0 \end{Bmatrix} \tag{A.23}$$

$$\{x\} = \begin{Bmatrix} q_0 \\ \gamma_{-1} \\ q_1 \\ \gamma_1 \\ \vdots \\ q_i \\ \gamma_i \\ \vdots \\ q_{n-1} \\ \gamma_{n-1} \\ q_n \\ \gamma_{n+1} \end{Bmatrix} \tag{A.24}$$

$$C = \begin{Bmatrix} \Delta^2 \dot{F}_{c0} + 2\Delta I_o \\ \Delta^2 \dot{G}_{c0} \\ \Delta^2 \dot{F}_{c1} \\ \Delta^2 \dot{G}_{c1} \\ \vdots \\ \Delta^2 \dot{F}_{cn-1} \\ \Delta^2 \dot{G}_{cn-1} \\ \frac{1}{2}\Delta^2 \dot{F}_{cn} - \Delta I_n \\ \Delta^2 \dot{G}_{cn} \end{Bmatrix} \tag{A.25}$$

$$\left.\begin{aligned} I_o &= \tfrac{1}{2} \int_{-1}^{1} V_2 \, d\xi \Big|_{\eta=0} \\ I_n &= \tfrac{1}{2} \int_{-1}^{1} V_2 \, d\xi \Big|_{\eta=\eta_n} \end{aligned}\right\} \qquad (A.26)$$

$$\left.\begin{aligned} q_{-1} &= q_1 + 2\Delta\left(-2 + \frac{E_\nu}{E_1} + T_o'\right) \\ q_{n+1} &= q_{n-1} + 2\Delta\left(2 - \frac{E_\nu}{E_1} + T_n'\right) \\ \dot{q}_{-1} &= \dot{q}_1 - 2\Delta I_o \\ \dot{q}_{n+1} &= \dot{q}_{n-1} + 2\Delta I_n \end{aligned}\right\} \qquad (A.27)$$

APPENDIX B

COMPUTER PROGRAM

```
 IBFTC MAIN
C      CREEP ANALYSIS OF CLAMPED CYLINDRICAL SHELL
C
       DIMENSION A(170),B(170),C(170),D(170),E(170)
       DIMENSION H(170),M(170),U(170),V(170),W(170)
       DIMENSION RHS(170),XYZ(23)
       DIMENSION SHEAR(85),GAMMA(85)
       DIMENSION U1(23),U2(23),V1(23),V2(23)
       DIMENSION NPHI(85),NPDOT(85)
       DIMENSION FODOT(85),GCDOT(85)
       DIMENSION LAMDX(85,23),LAMDT(85),LXDOT(85,23),LTDOT(85)
       DIMENSION SIGMX(85,23),SIGMT(85,23),SXDOT(85,23),STDOT(85,23)
       REAL NPHI,LAMDT,LAMDX,NPDOT,LTDOT,LXDOT
       REAL N,M
       COMMON/A1/N,M,U,V,W
       COMMON/A2/RHS,SHEAR,GAMMA
       COMMON/A3/NDIV,DELTA
       COMMON/A4/E1,E2,ENU
       COMMON/A5/SIGMX,SIGMT
       COMMON/A6/FODOT,GCDOT
       COMMON/A7/U1/A8/U2/A9/V1/A10/V2
       COMMON/A12/XINCR
       COMMON/PARA/ATZEE
C
C      DELTA          =LENGTH OF AN ELEMENT
C      NDIV           =NUMBER OF DIVISIONS ALONG LENGTH
C      INDEX          =EXPONENT OF POWER LAW
C      E1,E2,ENU      =ELASTIC CONSTANTS
C      FBS            =STRESS RATE AT STEADY STATE
C      AXX,ATHITA,ATZEE   =ANISOTROPIC COEFFICIENTS OF CREEP
C      FACTR          =FACTOR BY WHICH STRESS/STRESS RATE SHOULD BE DIVIDED
C                      TO GET INCREMENTAL TIME
C      ITMAX          =MAXIMUM NUMBER OF STEPS ALLOWED TO REACH STEADY STATE
C      IFIRST         =NUMBER OF STEPS UPTO WHICH ONLY STRESSES AND STRAINS
```

```
C                        ARE PRINTED WITHOUT GAP IN TIME
C     ILAST             =NUMBER OF STEPS UPTO WHICH ONLY STRESSES AND STRAINS
C                        ARE PRINTED WITH GAP IN TIME
C     INTER             =NUMBER OF STEPS TO BE SKIPPED WHILE PRINTING
C                        AFTER ILAST STEPS
C     ISTEP             =NODE UPTO WHICH STRESSES AND STRAINS ARE PRINTED
C                        WITHOUT GAP IN LENGTH
C     IGAP1             =STARTING NODE FOR PRINTING STRESSES ACROSS THICKNESS
C     IGAP2             =STARTING NODE FOR PRINTING STRESS RATES AND STRAIN RATE
C     IGAP3             =NUMBER OF NODES TO BE SKIPPED WHEN PRINTING STRESS
C                        RATES AND STRAIN RATES
C     IGAP4             =NUMBER OF NODES TO BE SKIPPED WHEN PRINTING STRESSES
C                        AND STRAINS
C
      READ 1,NDIV,ITMAX
      READ 1,INDEX
      READ 2,E1,E2,ENU
      READ 10,DELTA
      READ 10,AXX,ATHITA,ATZEE
      READ 10,EBS,FACTR
      READ 10,ALFA
      READ 1,IFIRST,ILAST,INTER,ISTEP
      READ 1,IGAP1,IGAP2,IGAP3,IGAP4
      READ 2,EMAX
      PRINT 3
      PRINT 4,NDIV
      PRINT 5,DELTA
      PRINT 6,EBS
      PRINT 7,FACTR
      PRINT 8,ITMAX
      PRINT 11,ALFA
      PRINT 9
      PRINT 29,E1,E2,ENU,INDEX,AXX,ATHITA,ATZEE
      PRINT 1,IFIRST,ILAST,INTER,ISTEP
      PRINT 1,IGAR1,IGAP2,IGAP3,IGAP4
      PRINT 2,EMAX
C
C     ELEMENTS OF MATRIX A ARE CALCULATED
C     DIAGONAL TERMS ARE STORED AS VECTORS A,B,C,D,E
C
```

```
      IPRINT =1
      INDEX=INDEX-1
      NUMB=NDIV*2+2
      NEW=NUMB-1
      DO 1100 I=1,NEW,2
      A(I)=1.
      B(I)=DELTA**2
      C(I)=-2.
      D(I)=0.
      E(I)=1.
      J=I+1
      A(J)=1.
      B(J)=0.
      C(J)=-2.
      D(J)=-DELTA**2
      E(J)=1.
 1100 CONTINUE
      A(2)=0.
      B(NEW)=0.
      C(2)=1.
      C(NEW)=1.
      C(NUMB)=1.
      D(2)=0.
      D(NUMB)=0.
      E(2)=0.
       E(3)=2.
      E(NEW)=0.
      E(NUMB)=0.
C
C     CALCULATION OF ELEMENTS OF MATRICES L AND U SUCH THAT L*U=A
C
      W(1)=0.
      V(1)=0.
      M(1)=0.
      N(1)=0.
      N(2)=0.
      ZMR1=0.
      ZNR=0.
      U(1)=C(1)
      ZUR=U(1)
```

```
      Z=B(1)
      ZMR=Z/ZUR
      M(2)=ZMR
      Z=A(1)
      ZNR1=Z/ZUR
      N(3)=ZNR1
      DO 200 I=2,NUMB
      W(I)=E(I)
      ZWR=W(I)
      Z=D(I)
      ZVR=Z-ZMR1*ZWR
      V(I)=ZVR
      Z=C(I)
      ZUR=Z-ZNR*ZWR-ZMR*ZVR
      U(I)=ZUR
      IF(I.EQ.NUMB)GO TO 200
      ZMR1=ZMR
      Z=B(I)
      ZMR=(Z-ZNR1*ZVR)/ZUR
      M(I+1)=ZMR
      ZNR=ZNR1
      IF(I.EQ.NEW)GO TO 200
      Z=A(I)
      ZNR1=Z/ZUR
      N(I+2)=ZNR1
  200 CONTINUE
C
C     ELASTIC SOLUTION
C     ITER          =COUNTER TO DETERMINE NUMBER OF STEPS
C
      ITER=1
      ISTOP=1
      PRINT 31
      DO 101 I=2,NUMB
      RHS(I)=0.
  101 CONTINUE
      RHS(1)=2.*DELTA*(2.-ENU/E1)/ALFA
      MEHRA=1
      CALL EQUAT(MEHRA,ALFA)
```

```
C
C     XIZER AND MEHRA ARE INTRODUCED TO MATCH SUBROUTINE REQUIREMENTS
C     NRING        =NUMBER OF DIVISIONS OF THICKNESS
C     IN STRESSES AND STRAINS AND THEIR RATES FIRST SUBSCRIPT DENOTES
C     NODE NUMBER AND SECOND SUBSCRIPT DENOTES POSITION ALONG THICKNESS
C
      NRING1=(INDEX+3)*2
      XINCR=2./FLOAT(NRING1)
      NRING=NRING1+1
      XX=E2/E1
      YY=ENU/E1
       ESTAR=E2-ENU**2/E1
      ZZ=SQRT(3.*ESTAR/E1)
      PP=E1/ESTAR
      QQ=E2/ESTAR
      RR=ENU/E2
      SS=(E1*AXX+ENU*ATZEE)/ESTAR
      TT=(E2*ATHITA+ENU*ATZEE)/ESTAR
      XKBAR1=(ENU*ATHITA+E1*ATZEE)/(E1*AXX+ENU*ATZEE)
      XKBAR2=(ENU*AXX+E2*ATZEE)/(E2*ATHITA+ENU*ATZEE)
      TODEL=2.*DELTA
      AA=2./ALFA
      BB=YY/ALFA
      CC=XX/ALFA
      M1=NDIV+2
      DO 102 I=2,M1
      NPHI(I)=AA-(SHEAR(I+1)-SHEAR(I-1))/TODEL
      LAMDT(I)=NPHI(I)-BB
      XZI=-1.
      DO 103 J=1,NRING
      LAMDX(I,J)=CC-YY*NPHI(I)-XZI*ZZ*(GAMMA(I+1)-GAMMA(I-1))/TODEL
      SIGMX(I,J)=(LAMDX(I,J)+YY*LAMDT(I))*PP
      SIGMT(I,J)=(LAMDT(I)+RR*LAMDX(I,J))*QQ
      XZI=XZI+XINCR
  103 CONTINUE
  102 CONTINUE
      MINOR=1
      GO TO 2001
 4002 TIME=0.
      PRINT 25,TIME
```

```
 4001 CALL DOTS(NRING,INDEX,AXX,ATHITA ,XKBAR1)
C
C     CREEP SOLUTION
C     LAST STEP (4001)IS INCLUDED IN BOTH ELASTIC AND CREEP SOLUTION
C     THIS STEP DETERMINES MODIFIED RHS FOR EQUATION A*Q=C
C
 1001 MINOR=2
      DO 201  J=1,NRING
      X=SIGMX(2,J)
      Y=SIGMT(2,J)
      XYZ(J)=V11(Y,X,INDEX,ATHITA,AXX)
  201 CONTINUE
      CALL QUAD(XYZ,NRING,XIZER)
      XIZER=XIZER/2.
      BEETA=DELTA**2
      RHS(1)=BEETA*FCDOT(2)+2.*DELTA*XIZER
      RHS(2)=BEETA*GCDOT(2)
      M1=2*NDIV+1
      L=3
      DO 202  I=3,M1, 2
      RHS(I)=FCDOT(L)*BEETA
      J=I+1
      RHS(J)=GCDOT(L)*BEETA
      L=L+1
  202 CONTINUE
      MEHRA=2
      CALL EQUAT(MEHRA,XIZER)
C
C     CALCULATION OF STRESS RATES AND STRAIN RATES
C
      M1=NDIV+2
      DO 301  I=2,M1
      NPDOT(I)=(SHEAR(I-1)-SHEAR(I+1))/TODEL
      DO 302 J=1,NRING
      X=SIGMX(I,J)
      Y=SIGMT(I,J)
      V2(J)=V11(Y,X,INDEX,ATHITA,AXX)
      V1(J)=V11(X,Y,INDEX,AXX,ATHITA)
      U1(J)=U11(X,Y,INDEX,AXX,ATHITA,XKBAR1)
      U2(J)=U11(Y,X,INDEX,ATHITA,AXX,XKBAR2)
```

```
  302 CONTINUE
      CALL QUAD(V2,NRING,TADD)
      CALL QUAD(V1,NRING,XADD)
      LTDOT(I)=NPDOT(I)+TADD/2.
      XZI=-1.
      DO 303 K=1,NRING
      LXDOT(I,K)=-YY*NPDOT(I)-XZI* ZZ*(GAMMA(I+1)-GAMMA(I-1))/(2.*DELTA
     1+XADD/2.
      SXDOT(I,K)=(LXDOT(I,K)+YY*LTDOT(I))*PP-SS*U1(K)
      STDOT(I,K)=(LTDOT(I)+RR*LXDOT(I,K))*QQ-TT*U2(K)
      XZI=XZI+XINCR
  303 CONTINUE
  301 CONTINUE
C
C     CALCULATION OF SUITABLE INCREMENT OF TIME
C
      DTIME=10.
      DO 401 I=2,M1
      DO 400 J=1,NRING
      IF(ABS(SIGMX(I,J)).LT.0.1)GO TO 400
      IF(ABS(SXDOT(I,J)).LT.EMAX)GO TO 400
      RATIO=SIGMX(I,J)/SXDOT(I,J)
      DTIME1=ABS(RATIO)
      IF(DTIME1.GT.DTIME)GO TO 400
      DTIME=DTIME1
  400 CONTINUE
  401 CONTINUE
      DTIME=DTIME/FACTR
      TIME=TIME+DTIME
      PRINT 23
      PRINT 25,TIME
      PRINT 1,ITER
C
C     CALCULATION OF STRESSES AND STRAINS AFTER DELTA TIME
C
      DO 501 I=2,M1
      LAMDT(I)=LAMDT(I)+DTIME*LTDOT(I)
      DO 501 J=1,NRING
      LAMDX(I,J)=LAMDX(I,J)+DTIME*LXDOT(I,J)
      SIGMX(I,J)=SIGMX(I,J)+DTIME*SXDOT(I,J)
```

```
      SIGMT(I,J)=SIGMT(I,J)+DTIME*STDOT(I,J)
  501 CONTINUE
C
C     CHECK FOR TERMINATION
C
      IF(ABS(SXDOT(2,NRING)).GT.EBS)GO TO 2001
      PRINT 26
      PRINT 32,ITER
      ISTOP=2
      MINI=3
      GO TO 601
 2001 IF(ITER.LE.IFIRST)MINI=1
      IF(ITER.GT.IFIRST)MINI=2
      IF(ITER.GT.ILAST)MINI=3
      IF(ITER.GT.ITMAX)GO TO 608
      ITER=ITER+1
      IPRINT=IPRINT+1
      IF(IPRINT.GE.INTER)IPRINT=0
      IF(MINI.NE.1.AND.IPRINT.NE.1)GO TO 4001
  601 PRINT 21
      DO 602 I=2,ISTEP
      PRINT22,I,SIGMX(I,1),SIGMX(I,NRING),SIGMT(I,1),SIGMT(I,NRING),
     1LAMDX(I,1),LAMDX(I,NRING),LAMDT(I)
  602 CONTINUE
      J=ISTEP+1
      DO 603 I=J,M1,IGAP4
      PRINT 22,I,SIGMX(I,1),SIGMX(I,NRING),SIGMT(I,1),SIGMT(I,NRING),
     1LAMDX(I,1),LAMDX(I,NRING),LAMDT(I)
  603 CONTINUE
      GO TO(4002,604),MINOR
  604 GO TO(4001,4001,605),MINI
  605 LL=M1/2-1
      PRINT 44
      DO 606 I=IGAP2,M1,IGAP3
      PRINT 45,I,(SXDOT(I,J),J=1,NRING,NRING1),(STDOT(I,J),J=1,NRING,
     1NRING1),(LXDOT(I,J),J=1,NRING,NRING1),LTDOT(I)
  606 CONTINUE
      DO 607 I=IGAP1,M1,LL
      PRINT 47,I,(SIGMX(I,J),J=1,NRING)
```

```
      PRINT 48,I,(SIGMT(I,J),J=1,NRING)
  607 CONTINUE
      GO TO(4001,609),ISTOP
  608 PRINT 27
  609 STOP
C
C     FORMAT SPECIFICATIONS
C
    1 FORMAT(10I5)
    2 FORMAT(7E13.6)
    3 FORMAT(*1INPUT DATA*//)
    4 FORMAT(10X,*TOTAL NUMBER OF DIVISIONS ALONG AXIS*,43X,*=*,I5)
    5 FORMAT(10X,*NON DIMENSIONAL LENGTH OF AN ELEMENT*,43X,*=*,F14.8)
    6 FORMAT(10X,*STRESS RATE ALLOWED AT STEADY STATE*,44X,*=*,F14.8)
    7 FORMAT(10X,*FACTOR BY WHICH STRESS/STRESS RATE SHOULD BE D9VIDED T
     1O GET INCREMENTAL TIME  =*,F14.8)
    8 FORMAT(10X,*MAXIMUM NUMBER OF STEPS ALLOWED TO STEADY STATE*,34X,
     1*=*,I5)
    9 FORMAT(/* PROPERTIES OF MATERIAL*//
     1*     MODULII  OF  ELASTICITY*,24X,*CREEP  LAW      ANISOTROPY
     2COEFFICIENTS*/*       AXIAL*,14X,*CIRCUMFRANTIAL       POISSON
     3          INDEX      AXIAL*,14X,*CIRCUMFRANTIAL      POISSON*/)
   10 FORMAT(7F14.6)
   11 FORMAT(10X,*MULTIPLYING FACTOR FOR REFRENCE STRESS*,37X,*=*,F12.5)
   21 FORMAT(* NODE  AXIAL STRESS                    TANGENTIAL STRESS
     1 AXIAL  STRAIN              TANG STRAIN  AXIAL MOMNT   TANGMOMNT*/
     2*        OUTER          INNER          OUTER          INNER          OUTER
     3          INNER*)
   22 FORMAT(1X,I4,2X,9E13.6)
   23 FORMAT(1X,120(1H-))
   25 FORMAT(20X,*TIME          =*,F15.6)
   26 FORMAT(/* THE SOLUTION HAS CONVERGED*//)
   27 FORMAT(/* THE SOLUTION DOES NOT CONVERGE*//)
   29 FORMAT(3(5X,E14.6),5X,I5,3(5X,E14.6))
   31 FORMAT(/* ELASTIC SOLUTION*//)
   32 FORMAT(/* TOTAL NUMBER OF ITERATIONS  =*,I5/)
   41 FORMAT(* INNER/OUTER,NODE,RATIO =*,2I5,F11.6)
   42 FORMAT(* RATIO =*,11F11.6)
   43 FORMAT(5F14.6)
```

```
   44 FORMAT(* NODE  AXIAL STRESS RATE*,10X,*CICUMFERENTIAL STRESS RATE*
     1,10X,*AXIAL STRAIN RATE*,10X,*CIRCUMFERERENTIAL STRAIN RATE*)
   45 FORMAT(1X,I4,2X,10E12.5)
   47 FORMAT(* AXIAL STRESS =*.I5,8E13.5/9E13.5/9E13.5)
   48 FORMAT(*   TAN STRESS =*,I5,8E13.5/9E13.5/9E13.5)
      END
 IBFTC EQUAT
      SUBROUTINE EQUAT(MEHRA,XIZER)
C
C     THIS SUBROUTINE DETERMINES THE VALUES OF SHEAR AND GAMMA
C     MEHRA=AN INDX WHICH SHOWS WHETHER THE PROBLEM IS ELASTIC OR
C     IT IS OF CREEP
C     XIZER=VALUE OF INTEGRAL IZERO (FOR CREEP ONLY)
C
      DIMENSION N(170),M(170),U(170),V(170),W(170)
      DIMENSION SHEAR(85),GAMMA(85)
      DIMENSION RHS(170)
      REAL N,M
      COMMON/A1/N,M,U,V,W
      COMMON/A2/RHS,SHEAR,GAMMA
      COMMON/A3/NDIV,DELTA
      COMMON/A4/E1,E2,ENU
      NUMB=2*NDIV+2
      CALLMTRIX(RHS,NUMB)
C
C      OUTPUT OBTAINED FROM SUBROUTINE IS STORED IN RHS ITSELF
C     NEXT STEP IS TO DISTRIBUTE VALUES OF RHS INTO GAMMA AND SHEAR
C                                                             VECTORS
      M1=NDIV+1
      J=3
      DO 101 I=3,M1
      SHEAR(I)=RHS(J)
      GAMMA(I)=RHS(J+1)
      J=J+2
  101 CONTINUE
      GO TO (102,103),MEHRA
  102 SHEAR(1)=SHEAR(3)-2.*DELTA*(2.-ENU/E1)/XIZER
      GO TO 104
  103 SHEAR(1)=SHEAR(3)-2.*DELTA*XIZER
  104 SHEAR(2)=RHS(1)
```

```
      GAMMA(1)=RHS(2)
      GAMMA(2)=0.
      MN=M1+1
      GAMMA(MN)=0.
      SHEAR(MN)=0.
      MN=MN+1
      SHEAR(MN)=RHS(NUMB-1)
      GAMMA(MN)=RHS(NUMB)
      RETURN
      END
 IBFTC DOTS
      SUBROUTINE DOTS(NRING,INDEX,AXX,ATHITA,XKBAR1)
C
C     THIS SUBROUTINE DETERMINES THE INTEGRALS GCDOT AND FCDOT
C     FROM THE KNOWN VALUES OF STRESSES
C
      DIMENSION SIGMX(85,23),SIGMT(85,23)
      DIMENSION FC(85),GC(85),FCDOT(85),GCDOT(85)
      DIMENSION U1(23),V2(23)
      COMMON/A3/NDIV,DELTA
      COMMON/A4/E1,E2,ENU
      COMMON/A5/SIGMX,SIGMT
      COMMON/A6/FCDOT,GCDOT
      COMMON/A7/U1/A10/V2
      COMMON/A12/XINCR
      COMMON/PARA/ATZEE
      TODEL=2.*DELTA
      FORDEL=4.*DELTA
      M=NDIV+2
      DO 101 I=2,M
      XZI=-1
      DO 102 J=1,NRING
      X=SIGMX(I,J)
      Y=SIGMT(I,J)
      U1(J)=U11(X,Y,INDEX,AXX,ATHITA,XKBAR1)
      U1(J)=U1(J)*XZI
      V2(J)=V11(Y,X,INDEX,ATHITA,AXX)
      XZI=XZI+XINCR
  102 CONTINUE
      CALL QUAD(V2,NRING,F)
```

```
      FC(I)=F
      CALL QUAD(U11,NRING,G)
      GC(I)=G
  101 CONTINUE
      E=E2-ENU**2/E1
      XM=1.5*SQRT(E2/2./E1)*(E1*AXX+ENU*ATZEE)/E
      FCDOT(2)=(FC(3)-FC(1))/TODEL
      FCDOT(M)=0.
      GCDOT(2)=XM*(GC(2)-GC(3))/DELTA
      GCDOT(M)=0.
      M1=M-1
      DO 103 I=3,M1
      FCDOT(I)=(FC(I+1)-FC(I-1))/FORDEL
      GCDOT(I)=XM*(GC(I-1)-GC(I+1))/TODEL
  103 CONTINUE
      RETURN
      END
 IBFTC V11
      FUNCTION V11(X,Y,INDEX,R,S)
C
C     EVALUATION OF CREEP STRAIN RATES FOR GIVEN STRESSES X,Y
C     IFX=AXIAL STRESS IT GIVES AXIAL STRAIN RATE
C     IF X=CIRCUMFRANTIAL STRESS IT GIVES CIRCUMFRANTIAL STRAIN RATE
C
      COMMON/PARA/ATZEE
      V11=SIGME(X,Y,INDEX,R,S)*(R*X+ATZEE*Y)
      RETURN
      END
 IBFTC U11
      FUNCTION U11(X,Y,INDEX,R,S,KBAR)
C
C     THIS FUNCTION DETERMINES U1 OR U2
C     IF X=AXIAL STRESS IT GIVES U1
C     IF X=CIRCUMFRANTIAL STRESS IT GIVES U2
C
      COMMON/PARA/ATZEE
      REAL KBAR
      U11=SIGME(X,Y,INDEX,R,S)*(X+KBAR*Y)
      RETURN
      END
```

```
 IBFTC SIGME
       FUNCTION SIGME(X,Y,INDEX,R,S)
C
C       DETERMINATION OF EFFECTIVE STRESS FOR GIVEN STRESSES X,Y
C
       COMMON/PARA/ATZE
       SIGME=SQRT(R*X**2+S*Y**2+2.*ATZE*X*Y)
       SIGME=SIGME**INDEX
       RETURN
       END
 IBFTC QUAD
       SUBROUTINE QUAD(FUNSN,N,RESLT)
C
C      EVALUATION OF INTEGRALS BY SIMPSON FORMULA
C      INPUT IS A SET OF VALUES OF FUNCTION
C
       DIMENSION FUNSN(23)
       COMMON/A12/XINCR
       RESLT=FUNSN(1)+FUNSN(N)
              M=N-3
       DO 101  I=2,M,2
       RESLT=RESLT+4.*FUNSN(I)
       J=I+1
       RESLT=RESLT+2.*FUNSN(J)
  101  CONTINUE
       RESLT=RESLT+4.*FUNSN(M+2)
       RESLT=RESLT*XINCR/3.
       RETURN
       END
 IBFTC MTRIX
       SUBROUTINE MTRIX(P,NUMB)
C
C      THIS SUBROUTINE FINDS SOLUTION OF AX=P
C      CHOLESKI UNSYMMETRICAL METHOD
C      OUTPUT VECTOR P IS STORED IN INPUT VECTOR P ITSELF
C      NUMB    -    =TOTAL NUMBER OF UNKNOWNS
C
       DIMENSION N(170),M(170),U(170),V(170),W(170)
       DIMENSION P(170)
       REAL N,M
```

```
      COMMON/AB/U,M,J,V,W
C
C     SOLUTION OF L*Y=P
C
      P(2)=P(2)-P(1)*M(2)
      DO 100 I=3,NUMB
      P(I)=P(I)-P(I-1)*M(I)-P(I-2)*J(I)
  100 CONTINUE
      NM=NUMB-1
      P(NUMB)=P(NUMB)/U(NUMB)
      P(NM)=(P(NM)-P(NUMB)*V(NUMB))/U(NM)
      DO 200 I=2,NM
      J=NM-I+1
      P(J)=(P(J)-R(J+1)*V(J+1)-P(J+2)*W(J+2))/U(J)
  200 CONTINUE
      RETURN
      END
 ENTRY
```

NOTE

OTHER PROGRAMS USED IN THIS WORK ARE

(1)CREEP ANALYSIS OF SIMPLY SUPPORTED CYLINDRICAL SHELL

(2)CREEP ANALYSIS OF CLAMPED SPHERICAL SHELL

(3)CREEP ANALYSIS OF CLAMPED CYLINDRICAL SHELL SUBJECTED TO UNIFORM PRESSURE AND AXISYMMETRIC TEMPERATURE FIELD

IN THESE PROGRAMS THOUGH THE EQUATIONS ARE DIFFRENT THE METHOD OF COMPUTATIONS IS SIMILAR TO THE ONE FOR CLAMPED CYLINDRICAL SHELL

DETAILS OF COMPUTER PROGRAM HAVE NOT BEEN INCLUDED,THEREFORE .